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Scattering of partially coherent electromagnetic beams by water droplets and ice crystals

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ABSTRACT

The conventional Lorenz–Mie theory is generalized for a case when the light source is partially spatially coherent. The influence of the degree of coherence of the incident field on the generalized Mueller matrix and the spectral degree of coherence of the scattered light is analytically studied by using the vector field instead of the scalar field to extend previous results on the angular intensity distribution. The results are compared with the Mueller matrix obtained from the Discrete Dipole Approximation (DDA) method, which is an average over an ensemble of stochastic incident beams. Special attention is paid to the Mueller matrix elements in the backward direction, and the results show some Mueller matrix elements, such as P_{22}, depend monotonically on the coherence length of the incident beam. Therefore, detecting back scattering Mueller matrix elements may be a promising method to measure the degree of coherence. The new formalism is applied to cases of large spherical droplets in water clouds and hexagonal ice crystals in cirrus clouds. The corona and glory phenomena due to spheres and halos associated with hexagonal ice crystals are found to disappear if the incident light tends to be highly incoherent.

1. Introduction

In the conventional theories of light scattering, such as Lorenz–Mie theory [1], Discrete Dipole Approximation (DDA) [2,3], and Finite-Difference Time-Domain (FDTD) method [4], the incident light is generally assumed to be fully coherent in both space and time. However, the assumption is not always justified. In reality, light acquires some degree of incoherence due to light source fluctuations or to interactions with random media such as a turbulent atmosphere. Although the general framework of optical coherence theory has long been well established [5–7], scarcely any attention has been paid to the effect of coherence on light scattering by deterministic media. The influence of spatial coherence on scattering by a particle was partially investigated by Cabaret et al. [8] and Greffet [9]. The extinction cross section of rotationally invariant scatterers was found not to depend on the transverse (or spatial) coherence length, but the extinction cross section of anisotropic scatterers did depend on the state of coherence of the illuminating field. Further research on the topic was conducted by van Dijk [10] and Fischer [11] who studied the effects of spatial coherence on the angular distribution of radiant intensity scattered by a sphere. By using the angular spectrum representation of a random field [6] and half wave expansion of the scattering amplitude, the intensity of a scattered field was analytically obtained. The angular distribution of radiant intensity depends strongly on the degree of coherence, but the extinguished power does not. Sukhov [12] numerically studied the effect of spatial coherence by a random medium on the properties of scattered fields, and the DDA method was used to demonstrate that the statistical properties of the scattered light from an inhomogeneous medium were altered due to coherence effects.

Previous theories have been limited to the study of the scattered field intensity, which does not fully describe a scatterer with respect to light scattering. A common

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generalization is to study the Mueller matrix, which relates the Stokes vector of the incident beam to the Stokes vector of the scattered beam. For a large class of light beams such as the so-called Gaussian Schell-model beam, the Mueller matrix for a sphere is derived analytically by extending the Lorenz–Mie theory. We developed a DDA code [3] by modifying the source field to incorporate incoherence, and the Mueller matrix was obtained by averaging over an ensemble of stochastic incident fields. In addition, we investigated the spectral degree of coherence of the scattered light. As applications of the formalism developed, we performed the computation of light scattering by water droplets and hexagonal ice crystals, which are the major scatterers inside atmospheric clouds. The coherence effects of spherical water droplets were studied by modifying the Lorenz–Mie theory. With the Invariant Imbedding *T*-matrix method (see [13] and references cited therein), we were able to study the coherence effects on halos of large hexagonal ice crystals.

2. Scattering of partially coherent light by a sphere

We consider the scattering of electromagnetic beams of an arbitrary state of spatial coherence by a homogenous sphere on the basis of a generalization of previous results in [10,11]. Note that the procedure is quite similar to that used by Lahiri and Wolf [14], where the refraction and reflection of partially coherent electromagnetic beams were considered.

2.1. Theory of coherence

We use some of the important results from the coherence theory of electromagnetic beams [7]. The stochastic nature of incoherent monochromatic light is represented by an ensemble of random fields $\{\mathbf{E}(\mathbf{r},\omega)\}$, where ω is the frequency, and for each realization, the field component is transverse to the direction of propagation. By choosing the same plane of reference as in the previous section, each random field can be expressed as

$$\mathbf{E}(\mathbf{r},\omega)=\begin{pmatrix}E_l(\mathbf{r},\omega)\\ E_r(\mathbf{r},\omega)\end{pmatrix}. \tag{1}$$

Following the same formalism used by Wolf [7], the second-order correlation properties of the stochastic field are fully characterized by the 2×2 cross-spectral density matrix (CSDM) defined by

$$\overline{\overline{W}}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\langle\mathbf{E}^*(\mathbf{r}_1,\omega)\cdot\mathbf{E}^T(\mathbf{r}_2,\omega)\rangle = \begin{pmatrix}\langle E_l^*(\mathbf{r}_1,\omega)E_l(\mathbf{r}_2,\omega)\rangle & \langle E_l^*(\mathbf{r}_1,\omega)E_r(\mathbf{r}_2,\omega)\rangle\\ \langle E_r^*(\mathbf{r}_1,\omega)E_l(\mathbf{r}_2,\omega)\rangle & \langle E_r^*(\mathbf{r}_1,\omega)E_r(\mathbf{r}_2,\omega)\rangle\end{pmatrix}, \tag{2}$$

where $\langle\,\rangle$ denotes the ensemble average. From the CSDM, the spectral density can be derived at point $\mathbf{r}$ and at frequency ω

$$S(\mathbf{r},\omega)=\mathrm{Tr}\overline{\overline{W}}(\mathbf{r},\mathbf{r},\omega), \tag{3}$$

which can be interpreted as a contribution to the intensity at point $\mathbf{r}$ from the field component of frequency ω. The spatial degree of coherence of the random field is defined by [7]

$$\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)=\frac{\mathrm{Tr}\overline{\overline{W}}(\mathbf{r}_1,\mathbf{r}_2,\omega)}{\sqrt{S(\mathbf{r}_1,\omega)}\sqrt{S(\mathbf{r}_2,\omega)}}, \tag{4}$$

which unifies both polarization and coherence. $0\le|\eta(\mathbf{r}_1,\mathbf{r}_2,\omega)|\le 1$ with 0 representing complete incoherence and 1 representing complete coherence.

2.2. Incident light

Incoherent beams can be represented by an ensemble of random fields and are realized using the angular spectrum representation [6]. A partially coherent beam propagating in the $+z$ direction can be expressed as

$$\mathbf{E}^{(i)}(\mathbf{r},\omega)=\int_{|\mathbf{u}'_\perp|<1}\mathbf{e}^{(i)}(\mathbf{u}'_\perp,\omega)\exp(ik\hat{\mathbf{u}}'\cdot\mathbf{r})\,d^2\mathbf{u}'_\perp, \tag{5}$$

where the coefficient $\mathbf{e}^{(i)}(\mathbf{u}'_\perp,\omega)$ is a two-component random vector defined by

$$\mathbf{e}^{(i)}(\mathbf{u}'_\perp,\omega)=\begin{pmatrix}e_l^{(i)}(\mathbf{u}'_\perp,\omega)\\ e_r^{(i)}(\mathbf{u}'_\perp,\omega)\end{pmatrix}, \tag{6}$$

$k=\omega/c$ is the wavenumber, $\hat{\mathbf{u}}'$ is the direction of propagation of each plane wave component, $\mathbf{u}'_\perp=(u'_x,u'_y)$ is the projection of $\hat{\mathbf{u}}'$ onto the $z=0$ plane, and the unit vector $\hat{\mathbf{u}}'$ points into the $z>0$ half space, i.e. $u'_z=\sqrt{1-|\mathbf{u}'_\perp|^2}$. The scattering plane is chosen as the reference plane.

The incident field is uniquely defined by the source at the $z=0$ plane, which is characterized by the CSDM

$$\overline{\overline{W}}^{(i)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\langle\mathbf{E}^{(i)*}(\boldsymbol{\rho}_1,\omega)\cdot\mathbf{E}^{(i)^T}(\boldsymbol{\rho}_2,\omega)\rangle, \tag{7}$$

where $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are 2D vectors in the $z=0$ plane. Using Eq. (5), we have

$$\overline{\overline{W}}^{(i)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) = \int d^2\mathbf{u}'_\perp\, d^2\mathbf{u}''_\perp\,\overline{\overline{\tilde{W}}}^{(i)}(\mathbf{u}'_\perp,\mathbf{u}''_\perp,\omega)\exp(-ik(\mathbf{u}'_\perp\cdot\boldsymbol{\rho}_1-\mathbf{u}''_\perp\cdot\boldsymbol{\rho}_2)), \tag{8}$$

where the angular correlation matrix $\overline{\overline{\tilde{W}}}^{(i)}(\mathbf{u}'_\perp,\mathbf{u}''_\perp,\omega)=\langle\mathbf{e}^{(i)*}(\mathbf{u}'_\perp)\cdot\mathbf{e}^{(i)T}(\mathbf{u}''_\perp)\rangle$ is defined as a four-dimensional Fourier transformation of the CSDM at the $z=0$ plane, i.e.

$$\overline{\overline{\tilde{W}}}^{(i)}(\mathbf{u}'_\perp,\mathbf{u}''_\perp,\omega) = \left(\frac{k}{2\pi}\right)^4\int d^2\rho_1\, d^2\rho_2\,\overline{\overline{W}}^{(i)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\exp(ik\mathbf{u}'_\perp\cdot\boldsymbol{\rho}_1-\mathbf{u}''_\perp\cdot\boldsymbol{\rho}_2)). \tag{9}$$

We consider a widely used class of partially coherent beams, the so-called Gaussian Schell-model beams, which have the following CSDM elements

$$W_{lm}^{(i)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=a_l a_m b_{lm}\exp\left(-\frac{\rho_1^2+\rho_2^2}{4\sigma_S^2}\right)\exp\left(-\frac{(\rho_1-\rho_2)^2}{2\sigma_\mu{}^2}\right). \tag{10}$$

The independently chosen parameter σ_S can be interpreted as the width of the beam, and σ_μ as the coherence length. The remaining parameters in Eq. (10) have the