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A Green's function formalism of energy and momentum transfer in fluctuational electrodynamics

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ABSTRACT

Radiative energy and momentum transfer due to fluctuations of electromagnetic fields arising due to temperature difference between objects is described in terms of the cross-spectral densities of the electromagnetic fields. We derive relations between thermal non-equilibrium contributions to energy and momentum transfer and surface integrals of tangential components of the dyadic Green's functions of the vector Helmholtz equation. The expressions derived here are applicable to objects of arbitrary shapes, dielectric functions, as well as magnetic permeabilities. For the case of radiative transfer, we derive expressions for the generalized transmissivity and generalized conductance that are shown to obey reciprocity and agree with theory of black body radiative transfer in the appropriate limit.

1. Introduction

Fluctuations of electromagnetic fields lead to thermal radiative transfer, via energy transfer, and van der Waals and Casimir forces, via momentum transfer. Diffraction and interference effects as well as tunneling of evanescent and surface waves, collectively known as near-field effects, are not taken into consideration by the classical theory of radiative transfer. Near-field effects become important when the length scale of importance becomes comparable to the characteristic thermal wavelength ($\lambda_T \approx 3000/T$ μm). For radiative transfer between two objects, an important length scale is the minimum inter-object spacing, l_{gap}. When $l_{gap} \ll \lambda_T$, tunneling of electromagnetic waves lead to enhancement of radiative transfer beyond the classical or far-field limit. Surface texturing, for instance by creating a periodic 1D or 2D pattern, introduces a length scale, l_p, that characterizes the period of the pattern. When $l_p \ll \lambda_T$, diffraction effects can lead to thermal emission patterns not usually associated with a planar surface [1].

It has been long recognized that near-field enhancement of radiative transfer due to surface polaritons can result in increased power density as well as efficiency [2–5]. However, this enhancement of energy transfer has not been used in any practical device, as yet, because of our inability to conceive of configurations other than two parallel surfaces with a thin vacuum gap in which an enhancement of similar magnitude occurs. Most investigations of near-field radiative transfer have been restricted to objects of few simple geometric shapes, each analyzed by a vector eigenfunction expansion method applicable to that geometry (planar geometry with vector plane waves [2,6–10], cylindrical surfaces with vector cylindrical waves [11], two spheres with vector spherical waves [12–15], sphere-plane with a combination of vector spherical and plane waves [16]). Even minor changes to the shape of the object can impose great challenges. Simulations of thermal emission from textured surfaces are usually performed using rigorous coupled wave analysis (RCWA) [17–19] or finite difference time domain (FDTD) methods [20], which are quite different from those used for simulations of near-field radiative transfer.

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Nomenclature

$\mathbf{E}$	electric field vector
$\underline{F}$	view factor
$\overline{\overline{G}}_e$	electric dyadic Green's function
$\overline{\overline{G}}_m$	magnetic dyadic Green's function
$\overline{\overline{G}}_E$	$\nabla \times \overline{\overline{G}}_e$
$\overline{\overline{G}}_M$	$\nabla \times \overline{\overline{G}}_m$
$\overline{\overline{G}}_o$	Green's function of contribution due to background or source radiation
$\overline{\overline{G}}^{(sc)}$	Green's function of contribution from waves scattered by interfaces
G^e	linearized conductance for radiative transfer
$\mathbf{H}$	magnetic field vector
$\overline{\overline{I}}$	identity matrix
$\mathbf{J}$	current density
T_l	temperature in object l
$\mathbf{P}$	poynting vector
Q	radiative heat transfer
$\tilde{R}_{hn}$	Fresnel reflection coefficients at interfaces between h and n
S_l	closed surface of object l
T^e	generalized transmissivity for radiative energy transfer
T^m	generalized transmissivity for momentum transfer
V_l	volume of object l
$\underline{V}_\delta$	volume of infinitesimal radius surrounding $\tilde{\mathbf{r}}$
$\overline{\overline{\mathcal{E}}}$	matrix of contribution to $\langle \mathbf{EE}^* \rangle_s$
$\overline{\overline{\mathcal{H}}}$	matrix of contribution to $\langle \mathbf{HH}^* \rangle_s$
$\overline{\overline{\mathcal{X}}}$	matrix of contribution to $\langle \mathbf{EH}^* \rangle_s$
c	speed of light
$\hbar$	reduced Planck's constant
k	wavevector
k_b	Boltzmann's constant
k_n	n component of wavevector $(n = x,y,z)$
k_{hz}	z component of wavevector in vacuum
k_ρ	$\sqrt{k_x^2 + k_y^2}$
l	thickness of vacuum gap
$\hat{\mathbf{n}}$	unit normal vector
$\mathbf{r}$	position vector of observation point
$\tilde{\mathbf{r}}$	position vector of source point
t	time
β	0 or 1
δ	delta function
$\boldsymbol{\rho}$	distance of points
ϵ	Levi-Civita symbol
ε	permittivity, $\varepsilon' + i\varepsilon''$
ε_o	permittivity of free space
Θ	energy of a photon at temperature T
μ	permeability, $\mu' + i\mu''$
μ_o	permeability of free space
$\underline{\nu}, \underline{\xi}$	1 or -1
$\overline{\overline{\sigma}}$	Maxwell stress tensor
ω	frequency
$\Re$	real part
$\Im$	imaginary part
Tr	trace

Superscripts

bb	blackbody
e	electric field
m	magnetic field
pp	planar–planar
(h)	vacuum
(l)	objects $(l = 1,2,\ldots,N)$
(p)	transverse magnetic
(s)	transverse electric
(μ)	polarization s or p
T	transpose
$*$	complex conjugate

Subscripts

h	vacuum
i, p, q	Cartesian components 1,2,3
l	objects $(l = 1,2,\ldots,N)$
s	symmetric summation
$1 \rightarrow 2$	from object 1 to 2

To design other types of surfaces that can exploit the enhancement, without posing the hurdles associated with two parallel surfaces, and also to design surfaces with new radiative properties by shape modification at nano/micro scale, we need a general method to predict all types of nanoscale effects on radiative transfer, irrespective of the size, shape or properties of the objects involved.

Kruger et al. [11,21] used fluctuational electrodynamics to develop a scattering matrix and operator formalism for computing non-equilibrium force and heat transfer interactions between objects with arbitrary shapes and frequency dependent dielectric permittivities. Biehs et al. [9] developed a formalism of nanoscale radiative transfer between two parallel surfaces similar to that of Landauer formalism of electron transport in mesoscopic devices [22–25]. Ben-Abdallah et al. [26] used Rytov's theory to develop a theoretical formalism for radiative transfer between many objects in the dipole limit. Messina et al. [27] proposed a scattering matrix version of nanoscale radiative transfer as well as dispersion forces that is valid for objects with arbitrary shapes as well as dielectric functions. Non-equilibrium fluctuational electrodynamical interactions between objects can be expressed in a scattering matrix formalism or in a Green's functions formalism, just as the electrical conductance for electron transport can be developed in terms of the scattering matrix or Green's function.

The work in this paper is an extension to a prior work published in this journal by one of the authors [28]. In Ref. [28], the focus was on the relation between cross-spectral densities of electromagnetic fields in thermal equilibrium and the dyadic Green's functions (DGFs) of the