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Optical forces in a non-diffracting vortex beam

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ABSTRACT

Optical force acting upon a dielectric microparticle illuminated by a non-diffracting vortex beam is expressed using the generalized Lorenz–Mie theory (GLMT). Numerical results are presented for different widths and topological charges of the vortex beam. We show that such particle may be stably trapped either in the dark center of the vortex beam, in one of the two stable positions placed off the optical axis, or as the third option it may circulate along almost circular trajectory having its radius smaller or equal to the radius of the smallest high intensity vortex ring.

1. Introduction

The force interaction between light and a microparticle results in an optical force which is studied and employed within the framework of optical micromanipulation techniques [1]. Optical tweezers [2], Raman tweezers [3], optical cell sorters [4], optical stretchers [5], or optical pikotenzometers [6] represent the most known examples of this interaction. The above-mentioned applications mainly use the transfer of the linear momentum of light from the Gaussian laser beams to an object. In the case of more complex spatial light distributions the particle behavior is strongly determined by its size with respect to the characteristic field pattern. This phenomenon is sometimes called "size-effect" and in the case of standing waves the particle is pushed with its center to the intensity maximum or minimum depending on its size [7,8]. However, particles of particular sizes are not pushed at all and the overall optical force is negligible. Such strong dependence of the optical force on the particle size has been employed in passive optical sorting of microparticles in one- and two-dimensional optical lattices [9–13] because particles of different sizes or compositions follow different trajectories across the optical lattice.

Except the linear momentum, light can posses also spin and orbital angular momentum [14]. Spin angular momentum is associated with the polarization of light [15] and its change, for example due to the light transmittance through a birefringent microobject, results in a torque rotating the microobject around its axis [16]. The orbital angular momentum is associated with the spatial field distribution in an optical vortex beams [14,17]. If an object is illuminated by such beam, the light scattering and consequent transfer of the linear and angular momentum from the vortex beam to an object leads to the rotation of absorbing particle in the dark center of the vortex beam [18,19] or orbiting of microparticles around the beam axis in the high-intensity ring of the vortex beam [20–32].

In this paper, we focused on the spherical particles of micrometer sizes illuminated by vortex Bessel beam of varying beam width and topological charge. We used the generalized Lorenz–Mie theory [33–42] to calculate the optical forces acting upon a single particle and to investigate the particle behavior. In coincidence with the

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previous experimental observations we found that for certain conditions the particle orbits around the beam axis in a circle having radius smaller or equal to the radius of the first bright vortex ring. However, we also found that non-absorbing particles of certain sizes may be stably trapped either at the dark spot placed on the vortex beam axis or at one of the two azimuthal positions placed off the optical axis.

2. Theoretical Bessel beam description

An ideal Bessel beam is formed by an interference of the conical bunch of plane waves that all propagate with their $\mathbf{k}$ vectors tilted by an angle α_0 toward the optical axis z [43]. Since all the plane waves have equal axial component k_z of their wave vector $\mathbf{k}$, the angular spectrum of the plane waves of this beam is described by the delta function $\delta(\alpha-\alpha_0)\exp(im\phi)$ where α is the polar angle, m is an integer number called the topological charge of the vortex beam and ϕ is the azimuthal angle. The interference between all the plane waves results in the radial electric field distribution given by the Bessel function of the first kind and the m-th order $J_m(k_r\rho)$, where $\rho=\sqrt{x^2+y^2}$ is the radial distance from z-axis and $k_r=k\sin(\alpha_0)$ is the radial component of the wave vector $\mathbf{k}$. Since the Bessel beam belongs among the group of propagation invariant optical fields (also called non-diffracting beams) [44], it keeps the lateral intensity profile unvarying while propagating along the z-axis. In the case of the zero-order Bessel beam ($m=0$) all the plane waves interfere on the optical axis in phase and, consequently, an intensity maximum is created here. In the case of a higher order Bessel beam ($|m|>0$) all the plane waves are phase shifted with respect to each other in such a way that the phase shift around the cone is equal to $2\pi m$. Since the plane waves with all possible phase shifts interfere on the optical axis, such a destructive interference creates the intensity minimum along the optical axis and, therefore, the doughnut or optical vortex beam is generated.

The ideal Bessel beam would be of infinite transverse extent and carry infinite amount of energy. Therefore, in reality only an approximation to this ideal case, called quasi-Bessel beam (QBB), can be obtained over a limited axial distance [45]. There are an increasing number of papers dealing with theoretical aspects of QBBs and attempting to tune their properties closer to those of an ideal non-diffracting beams (see for example Ref. [46] and references therein). We focus here on the Bessel beams generated by an axicon [47–51]. For the theoretical calculations let us assume the ideal Bessel beam which is formed behind the axicon illuminated by the plane wave linearly polarized along the x-axis. The vector electric field of such Bessel beam can be described as [52,53]

$$\mathbf{E}(\rho,\phi,z)=E_0(\alpha_0)e^{ikz\cos\alpha_0}(-i)^m e^{im\phi}$$
$$\times\left(\left\{J_m(k_r\rho)+\frac{1}{2}[J_{m+2}(k_r\rho)e^{2i\phi}+J_{m-2}(k_r\rho)e^{-2i\phi}]\frac{1-\cos\alpha_0}{1+\cos\alpha_0}\right\}\mathbf{e}_x\right.$$
$$+\frac{1}{2i}[J_{m+2}(k_r\rho)e^{2i\phi}-J_{m-2}(k_r\rho)e^{-2i\phi}]\frac{1-\cos\alpha_0}{1+\cos\alpha_0}\mathbf{e}_y$$
$$\left.-i[J_{m+1}(k_r\rho)e^{i\phi}-J_{m-1}(k_r\rho)e^{-i\phi}]\frac{\sin\alpha_0}{1+\cos\alpha_0}\mathbf{e}_z\right), \quad (1)$$

where ϕ is the azimuthal angle and $\mathbf{e}_{x,y,z}$ are the base vectors along x, y, and z Cartesian coordinate axes.

For practical reasons we define the beam width ρ_m of the vortex beam with $|m|>0$ using the radius of the first maximum of the intensity in the radial direction, obtained from $\mathrm{d}J_m(k_r\rho_m)/\mathrm{d}\rho_m=0$. The beam width ρ_m can be related to the radius ρ_0 of the first intensity minimum in the radial direction of the zeroth order Bessel beam using $J_0(k_r\rho_0)=0$. For the topological charges up to 3 we obtain $\rho_1=0.7656\rho_0$, $\rho_2=1.27\rho_0$, $\rho_3=1.747\rho_0$ where [38]

$$\rho_0=\frac{2.4048}{k_r}=\frac{2.4048}{k\sin(\alpha_0)}. \quad (2)$$

The power carried by the BB core (for $m=0$) or the innermost high intensity BB ring (for $m=\pm1,\pm2,\ldots$) can be expressed in the paraxial case [38] as

$$P_{m,\mathrm{core}}\simeq-\frac{\pi k E_0^2\rho_0^2}{2\omega\mu_0}\frac{\sigma_m^2}{\sigma_0^2}J_{m-1}(\sigma_m)J_{m+1}(\sigma_m), \quad (3)$$

where ω is the light angular frequency, μ_0 is the vacuum permeability, σ_m is the first off-axis root of the Bessel function of the m-th order (e.g. $\sigma_0=2.4048$, $\sigma_1=3.8317$, $\sigma_2=5.1356$, etc.). The minus sign in Eq. (3) is compensated by opposite signs of $J_{m-1}(\sigma_m)$ and $J_{m+1}(\sigma_m)$. Therefore, the power carried by the central ring of the BB of topological charge $m=1$ is $1.53\times$ bigger than the power carried by the zeroth order BB. In the case of higher topological charges we obtain $P_2/P_0=1.95$, $P_3/P_0=2.32$.

For all the calculations presented in this paper we determine the value of the electric field intensity $E_0(\alpha)$ from the same power $P_{0,\mathrm{core}}=5$ mW carried by the central core of the zeroth order Bessel Beam independently on the core radius.

3. Optical force

To express the optical force acting upon a spherical dielectric particle placed into the BB we use the Lorenz–Mie approach [33,37,38]. Inspired by the work of Taylor [54] we have expressed the scattered field coefficients A_{ln} and B_{ln} analytically for the vortex BB. It has shortened the computation time in the Matlab environment tremendously.

3.1. Analytical form of the scattering coefficients

Let us consider a spherical particle of radius a placed into electromagnetic field having an electric field intensity given by Eq. (1), i.e. into the optical vortex of topological charge m. The sphere is located with its center in Cartesian coordinates x',y',z' (corresponding cylindrical coordinates are ρ',ϕ',z'). Following the classical approach of the light scattering by a spherical object, the scattered field can be expressed using the expansion into the spherical harmonic functions Y_{ln}. The coefficients A_{ln} and B_{ln} of such expansion for electric field and magnetic fields of the higher order Bessel beam, respectively,