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Phase contrast metrology using digital in-line holography: General models and reconstruction of phase discontinuities

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ABSTRACT

General models are developed to describe in-line digital phase contrast experiments. They link analytically the reconstruction process and the different parameters of the set-up. Reconstructions of phase discontinuities are realized using 2 Dimensional-Fractional Fourier Transforms (2D-FRFT). They give the 3D position and the dimension of the phase object, and the phase shift introduced. Experimental results and simulations are in good concordance. The technique can be used with monochromatic or with large spectrum laser sources.

1. Introduction

Digital holography has important potentiality for the characterization of particles, powders and flows, and the number of applications has grown intensively in the last decade (see for example [1,2]). In this field of research, the elaboration of scalar diffraction models that can describe a whole imaging set-up is particularly attractive for the realization of in situ measurements. Using a generalized Huygens-Fresnel formalism, we could describe complex systems and perform measurements of microflows in micropipes [3]. Unfortunately, the differentiation of absorbing objects from purely phase objects remains difficult. Digital in-line holography gives classically the diameter of the object and its 3D position. In phase contrast experiments, a third fundamental parameter needs to be determined: the phase shift introduced by the object. Thus, much work has to be done to distinguish precisely opaque objects from purely transparent and more generally from partially transparent phase objects. The analysis of phase objects using digital phase contrast has received much attention since pioneer work of [4]. Recently, we could propose an in-line configuration, with original reconstruction of the phase objects using 2 Dimensional Fractional-order Fourier Transformations (2D-FRFT) [5]. To extend the domain of applications of this technique, the elaboration of global models that can describe any set-up represents an important challenge. It should give the opportunity to predict a wide variety of systems: from configurations in microscopy to the visualization through anamorphic systems as cylindrical pipes or micropipes. Such models have been developed in Continuous Wave (CW) and femtosecond regimes for the reconstruction of circular opaque objects in digital holography [6,7]. We present in this work the extension of these formalism based on generalized Huygens-Fresnel integrals to the description of digital phase contrast experiments. The imaging system will be described in terms of transfer matrices, in order to describe any in-line configuration. Different regimes of operation will be described: from CW regime in Sections 2 and 3, to sources with large spectrum as femtosecond lasers in Section 4. Our models will link analytically the process of reconstruction (fractional orders of the transforms to be applied, magnification factors) to the different parameters of the set-up (nature and position of the optical elements)

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and the characteristics of the phase object under investigation. The phase objects that we will consider are made of transparent, homogeneous, isotropic materials, and the polarization of the incident laser light has no influence on the diffraction patterns produced. We will thus consider a scalar model to describe the propagation of the electric field. Experimental results will be presented to validate the models. The potentiality of the method for the reconstruction of transparent phase objects through cylindrical systems, or the reconstruction of non-spherical objects will be further detailed.

2. Digital phase contrast in CW regime

2.1. Expression of the diffracted field in the plane of the CCD sensor

We consider the general configuration of Fig. 1. A monochromatic Gaussian beam (whose waist dimension is w_0 in the plane $z=0$) propagates through the first part of a general system along the z-axis. This part is described by transfer matrices M_1^x and M_1^y, along both transverse x- and y-axes. The beam is then diffracted by a transparent phase object. This dephasing area will consist in a circular thin film of ITO (Indium Tin Oxide) of diameter D, deposited on a silica substrate, in the experiments. The phase shift introduced at the wavelength λ is noted φ. The losses (absorption) induced by the diffracting element are negligible. The diffracted beam propagates then through the second part of the system (from the phase object to the CCD sensor, described by transfer matrices M_2^x and M_2^y). Let us establish the expression of the diffracted field in the plane of the CCD sensor. The first step consists in calculating the expression of the field that is diffracted in the plane where the diffracting element is located ($z=\ell_1$). It is calculated using the generalized Huygens-Fresnel integral [8–10, 6]:

$$E_1(\xi,\eta,\ell_1,\omega)=\frac{\omega\exp\left(i\frac{\omega}{c}n_{\ell_1}\ell_1\right)}{i2\pi c\sqrt{B_1^xB_1^y}}\int_{\mathbb{R}^2}E_0(x,y,0,\omega)\times\exp\left[i\frac{\omega}{2cB_1^x}(A_1^xx^2-2\xi x+D_1^x\xi^2)\right]\times\exp\left[i\frac{\omega}{2cB_1^y}(A_1^yy^2-2\eta y+D_1^y\eta^2)\right]dx\,dy, \quad (1)$$

where ξ and η are the transverse coordinates in the plan of the phase object, $E_0(x,y,0,\omega)$ is the electric field of the incident Gaussian beam, $n_{\ell_1}\ell_1$ is the optical pathway, and the different A_1^p, B_1^p, C_1^p, D_1^p (with $p=x$ or $p=y$) are the coefficients of the transfer matrix between the plane of the beam waist ($z=0$) and the plane of the diffracting element ($z=\ell_1$). This approach allows to describe cylindrical geometries introducing different matrices along the x- and y-directions. The second step consists in calculating the field diffracted by the phase object in the plane where the image is recorded (on the CCD sensor). In a similar way, it is obtained by the following integral:

$$E(x',y',\ell_1+\ell_2,\omega)=\frac{\omega\exp\left(i\frac{\omega}{c}n_{\ell_2}\ell_2\right)}{i2\pi c\sqrt{B_2^xB_2^y}}\int_{\mathbb{R}^2}(1+(e^{i\varphi}-1)\times T(\xi,\eta))E_1(\xi,\eta,\ell_1,\omega)\times\exp\left[i\frac{\omega}{2cB_2^x}(A_2^x\xi^2-2x'\xi+D_2^xx'^2)\right]\times\exp\left[i\frac{\omega}{2cB_2^y}(A_2^y\eta^2-2y'\eta+D_2^yy'^2)\right]d\xi\,d\eta, \quad (2)$$

where D is the diameter of the phase object, and where the function $T(\xi,\eta)$ is a circular disk function in the case of a circular phase object which equals 1 within a disk of radius $D/2$ and 0 otherwise. Expanding $T(\xi,\eta)$ over a basis of Gaussian functions (see Appendix A), the integral of Eq. (2) gives after evaluation:

$$E(x',y',\ell_1+\ell_2,\omega)=\frac{\omega\exp\left(i\frac{\omega}{c}(n_{\ell_1}\ell_1+n_{\ell_2}\ell_2)\right)}{i2\pi c\sqrt{B_2^xB_2^y}}\times(R(x',y',\ell_1+\ell_2,\omega)+(e^{i\varphi}-1)\times O(x',y',\ell_1+\ell_2,\omega)). \quad (3)$$

R is the field that would be obtained in the plane of the CCD sensor in the absence of the phase plate. O is the field that would be transmitted by a circular aperture of radius $D/2$. These terms can be expressed as functions of the different coefficients A_2^p, B_2^p, C_2^p, D_2^p (with $p=x$ or $p=y$) that are the coefficients of the transfer matrices between the plane where the diffracting element is located and the plane of the CCD sensor, and the different characteristics of the incident beam and the phase plate (see Appendix A and references therein).

2.2. Principle of digital reconstruction by fractional Fourier transformation

The mathematical definition of the 2D-FRFT is given in Refs. [11–13]. We consider the 2D-FRFT of orders α_x along the x-axis and α_y along the y-axis of a 2D-function $I(x,y)$ (i.e. an image). The exact formulation that we use can be found in Appendix B and in Refs. [6,5]. The kernel of this

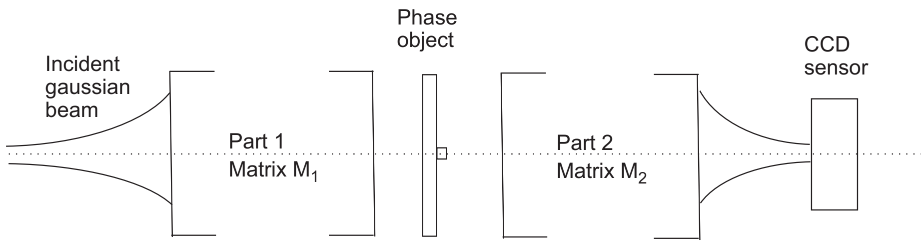

Fig. 1. Experimental set-up.