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Electromagnetic scattering by a bounded obstacle in a parallel plate waveguide

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A R T I C L E I N F O

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A B S T R A C T

This paper concerns scattering of an electromagnetic wave by a bounded object located inside a parallel plate waveguide. The exciting field in the waveguide is either an arbitrary source located at a finite distance from the obstacle or a plane wave generated in the far zone. In the latter case, the generating field corresponds to the lowest propagating mode (TEM) in the waveguide. The analytic treatment of the problem relies on an extension of the null field approach, or the T-matrix method, originally developed by Peter Waterman, and later generalized to deal with objects close to an interface. The present paper generalizes this approach further to deal with obstacles inside a parallel plate waveguide. This problem shows features that reflect both the two-dimensional geometry and the three-dimensional scattering characteristics. The analysis is illustrated by several numerical examples.

1. Introduction

Recent theoretical progress in the development of useful scattering identities – sum rules [1–4] – have initiated several attempts to verify these identities experimentally [4–9]. These sum rules relate the dynamical behavior of the scattering and absorption behavior of the scatterer to the static properties of the scatterer (polarizability dyadics).

The scattering identities have successfully been verified in free space [5–8]. In many respects, the parallel plate waveguide shows a more controlled environment for these measurements. Initial investigations show that this geometry is accessible [4,9]. A detailed investigation of the static properties of an obstacle between two parallel plates has also been reported recently [10].

The complexity of the solution increases dramatically if an obstacle is introduced in the waveguide. In the vicinity of the scatterer, the scattered field behaves as a solution of a three-dimensional scattering problem. However, far away from the scatterer, the field does not decline as $1/r$, as it does for a three-dimensional problem, but vanishes as $1/\sqrt{\rho}$, where the distance to the vertical axis is denoted ρ. Nevertheless, far away from the scatterer, the problem is still a three-dimensional scattering problem, since there are variations in the fields in the vertical direction. Only at frequencies below the first cutoff frequency, defined by $k_0 d = \pi$, where k_0 is the wave number in vacuum, and d is the distance between the plates, the problem is two-dimensional, in that there are no variations in the vertical direction of the fields below this frequency.

The presence of the parallel plates is usually solved by the introduction of an appropriate Green's dyadic [11]. However, in this paper, we do not pursue this line of solution technique further. Instead, we use the free space Green's dyadic, and solve the problem with parallel plates and scatterer simultaneously. The entire solution employs the integral representation of the solution. This integral representation approach to solve the scattering problem was originally introduced by Peter Waterman [12], and it has proven to be a very powerful and useful technique to solve a large variety of scattering problems, not only

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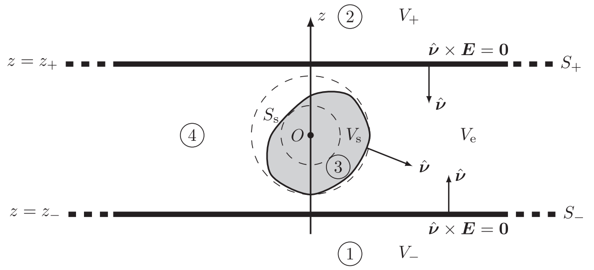

Fig. 1. The geometry of the direct scattering problem with two perfectly conducting planes S_+ and S_- and a scatterer with bounding surface S_s.

electromagnetic, but also acoustic and elastodynamic problems. In fact, the present geometry is an extension of the results with buried obstacle close to a planar interface—layered or not [13–20]. Similar technique to solve the electromagnetic scattering problem by obstacles inside a cylindrical waveguide has also been reported [21].

The results presented in this paper are inclined toward microwave applications. There are, however, no such limitations in the results. The technique applies equally well to applications at higher frequencies, *e.g.*, THz and IR, such as the computation of the scattering effects of impurities in thin films *etc*.

2. Formulation of the problem

A finite scatterer with bounding surface S_s defines the region V_s. Two infinite, perfectly conducting planes, S_+ and S_-, confine the two disjoint regions V_e and V_s, see Fig. 1. These planes are parameterized by $z=z_+$ and $z=z_-$, respectively, and without loss of generality, it is assumed that $z_+>0$ and $z_-<0$. The location of the origin O is arbitrary, but it is important for the analysis that it is located somewhere in V_s. The regions above S_+ and below S_- are denoted by V_+ and V_-, respectively. The sources of the problem are assumed to be located in $V_i\subset V_e$, between the surfaces S_+ and S_-.

To proceed, the time-harmonic electric and magnetic fields satisfy the free-space Maxwell equations in V_e (we use the time convention $\exp\{-\mathrm{i}\omega t\}$),

$$\begin{cases}\nabla\times\boldsymbol{E}(\boldsymbol{r})=\mathrm{i}k_0\eta_0\boldsymbol{H}(\boldsymbol{r}) \\ \nabla\times\eta_0\boldsymbol{H}(\boldsymbol{r})=-\mathrm{i}k_0\boldsymbol{E}(\boldsymbol{r})\end{cases}\quad \boldsymbol{r}\in V_e \tag{1}$$

where $k_0=\omega/c_0$ and η_0 are the angular wave number and wave impedance in free space, respectively. The boundary conditions on the bounding surfaces are

$$\begin{cases}\hat{\boldsymbol{z}}\times\boldsymbol{E}(\boldsymbol{r})=\boldsymbol{0}, & \boldsymbol{r}\in S_+\cup S_- \\ \boldsymbol{\nu}\times\boldsymbol{E}(\boldsymbol{r})=\boldsymbol{0}, & \boldsymbol{r}\in S_s\end{cases} \tag{2}$$

The scatterer V_s is here assumed to be a perfectly conducting body. This assumption can easily be relaxed, see below. With an appropriate radiation condition in V_e at large lateral distances, Eq. (1) has a unique solution.

2.1. Integral representation of the solution

Let $\boldsymbol{E}_i$ denote the incident electric field with sources located in V_e, and define the scattered electric field as $\boldsymbol{E}_s=\boldsymbol{E}-\boldsymbol{E}_i$. The incident field $\boldsymbol{E}_i$ is the field with no obstacle or plates present. With the directions of the unit normals defined as in Fig. 1, the solution of (1) and (2) satisfies the surface integral representation [22]

$$-\frac{1}{\mathrm{i}k_0}\nabla\times\left(\nabla\times\iint_{S_+\cup S_-\cup S_s}\mathbf{G}_e(k_0,|\boldsymbol{r}-\boldsymbol{r}'|)\cdot\boldsymbol{K}(\boldsymbol{r}')\,\mathrm{d}S'\right)=\begin{cases}\boldsymbol{E}_s(\boldsymbol{r}), & \boldsymbol{r}\in V_e \\ -\boldsymbol{E}_i(\boldsymbol{r}), & \boldsymbol{r}\in V_+\cup V_-\cup V_s\end{cases} \tag{3}$$

where $\boldsymbol{K}=\boldsymbol{\nu}\times\eta_0\boldsymbol{H}$, and the electric Green's dyadic

$$\mathbf{G}_e(k_0,|\boldsymbol{r}-\boldsymbol{r}'|)=\left(\mathbf{I}_3+\frac{1}{k_0^2}\nabla\nabla\right)\frac{\mathrm{e}^{\mathrm{i}k_0|\boldsymbol{r}-\boldsymbol{r}'|}}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}$$

The integral representation also contains a surface integral evaluated at large lateral distances, but proper radiation conditions at large lateral distances make this integral vanish. This surface integral representation is the starting point in the null-field approach.

3. Basis functions and expansions

3.1. Spherical vector waves

We introduce the out-going or radiating spherical vector waves, $\boldsymbol{u}_n(k\boldsymbol{r})$, defined as [23] ($\boldsymbol{u}_{\tau\sigma ml}(k\boldsymbol{r})=\boldsymbol{u}_{\tau n}(k\boldsymbol{r})=\boldsymbol{u}_n(k\boldsymbol{r})$)

$$\begin{cases}\boldsymbol{u}_{1n}(k\boldsymbol{r})=h_l^{(1)}(kr)\boldsymbol{A}_{1n}(\hat{\boldsymbol{r}}) \\ \boldsymbol{u}_{2n}(k\boldsymbol{r})=\dfrac{(krh_l^{(1)}(kr))'}{kr}\boldsymbol{A}_{2n}(\hat{\boldsymbol{r}})+\sqrt{l(l+1)}\dfrac{h_l^{(1)}(kr)}{kr}\boldsymbol{A}_{3n}(\hat{\boldsymbol{r}})\end{cases}$$

and regular spherical vector waves $\boldsymbol{v}_{\tau\sigma ml}(k\boldsymbol{r})$ as

$$\begin{cases}\boldsymbol{v}_{1n}(k\boldsymbol{r})=j_l(kr)\boldsymbol{A}_{1n}(\hat{\boldsymbol{r}}) \\ \boldsymbol{v}_{2n}(k\boldsymbol{r})=\dfrac{(krj_l(kr))'}{kr}\boldsymbol{A}_{2n}(\hat{\boldsymbol{r}})+\sqrt{l(l+1)}\dfrac{j_l(kr)}{kr}\boldsymbol{A}_{3n}(\hat{\boldsymbol{r}})\end{cases}$$

where $j_l(kr)$ and $h_l^{(1)}(kr)$ denote the spherical Bessel functions and the spherical Hankel functions of the first kind, respectively, and where the vector spherical harmonics are denoted $\boldsymbol{A}_{\tau\sigma ml}(\hat{\boldsymbol{r}})=\boldsymbol{A}_{\tau n}(\hat{\boldsymbol{r}})=\boldsymbol{A}_n(\hat{\boldsymbol{r}})$, $\hat{\boldsymbol{r}}=\boldsymbol{r}/|\boldsymbol{r}|$. The index n is a multi-index that consists of three or four different indices, *i.e.*, $n=\sigma ml$ or $n=\tau\sigma ml$, depending on the context, where $\tau=1,2,3$, $\sigma=\mathrm{e},\mathrm{o}$, $m=0,1,2,\ldots,l$, and $l=1,2,3,\ldots$. Their definitions are

$$\begin{cases}\boldsymbol{A}_{1n}(\hat{\boldsymbol{r}})=\dfrac{1}{\sqrt{l(l+1)}}\nabla\times(\boldsymbol{r}Y_n(\hat{\boldsymbol{r}}))=\dfrac{1}{\sqrt{l(l+1)}}\nabla Y_n(\hat{\boldsymbol{r}})\times\boldsymbol{r} \\ \boldsymbol{A}_{2n}(\hat{\boldsymbol{r}})=\dfrac{1}{\sqrt{l(l+1)}}r\nabla Y_n(\hat{\boldsymbol{r}}) \\ \boldsymbol{A}_{3n}(\hat{\boldsymbol{r}})=\hat{\boldsymbol{r}}Y_n(\hat{\boldsymbol{r}})\end{cases}$$

The spherical harmonics, $Y_n(\hat{\boldsymbol{r}})=Y_{\sigma ml}(\theta,\phi)$, are defined by [23]

$$Y_{\sigma ml}(\theta,\phi)=C_{lm}P_l^m(\cos\theta)\begin{Bmatrix}\cos m\phi \\ \sin m\phi\end{Bmatrix}$$