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Linearization of a scalar matrix operator method radiative transfer model with respect to aerosol and surface properties

Suniti V. Sanghavi *, John V. Martonchik, Anthony B. Davis, David J. Diner

Jet Propulsion Laboratory, Caltech, 4800 Oak Grove Dr, Pasadena, CA 91109, USA

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A B S T R A C T

In this paper, we review the radiative transfer formalism of the matrix operator method, and present the analytic form for its differentiation with respect to aerosol optical thickness, microphysical parameters and surface parameters. This "linearization" is an exact method that allows for an accurate and speedy computation of the Jacobian matrix, which is key to most optimization-based retrieval methods. We define an aerosol in terms of its optical thickness, complex refractive index and lognormal size distribution. We consider a bimodal aerosol distribution, consisting of a fine and coarse mode, such that the two modes also differ in their respective complex refractive indices. Three types of surfaces have been considered, viz. a purely Lambertian surface, a modified Rahman–Pinty–Verstraete bidirectional reflecting surface, and a Fresnel-reflecting ocean surface. We verify our results by comparing our linearized Jacobians of normalized intensities calculated at four different wavelengths in the visible (VIS) and near-infrared (NIR) and viewing angles ranging from -75° through 0° to 75° with those computed by the method of finite differences. We demonstrate the guaranteed accuracy of the linearized approach by contrasting it with the finite difference method which can only be used as a rough estimate due to its sensitivity to step size, especially for derivatives with respect to aerosol microphysical parameters. We also report that the computational speed-up due to linearization improves with the number of parameters involved, parity being achieved with the finite difference method for just one parameter. Finally, we discuss the features of the illustrated Jacobians as a function of viewing angle and wavelengths.

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1. Introduction

This work is motivated by a need to find a speedy and accurate method for computing the Jacobian matrix necessary for an optimized retrieval of aerosol and surface parameters from a multi-angular, multispectral satellite instrument like the Multi-angle Imaging SpectroRadiometer (MISR) [6]. Consequently, we show our results mainly for a multi-angular satellite viewing geometry in the VIS–NIR spectral regions corresponding to the MISR instrument.

Linearization with respect to scattering particles has already been applied to radiative transfer methods based on discrete ordinates [28,27], the Gauss–Seidel method [13] and recently the Markov chain formalism [4]. This work is the first time this approach as been applied to the matrix operator method, which, according to Lenoble [17] is especially suited to the simulation of scattering atmospheres.

We first present in Section 2 a review of the radiative transfer equation and its formulation in the framework of the matrix operator method (MOM) [9–11,15,14,18]. We have improved an existing model [19] and extended it to "linearize" or analytically compute the derivative of the forward model with respect to aerosol and surface parameters. Our model will henceforth be referred to as smartMOM

* Corresponding author. Tel.: +1 818 354 2500.
E-mail address: Suniti.V.Sanghavi@jpl.nasa.gov (S.V. Sanghavi).

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(**s**imulated **m**easurement of the **a**tmosphere using **r**adiative **t**ransfer based on the **M**atrix **O**perator **M**ethod).

In Section 3 we briefly define the Jacobian matrix, then proceed to present the linearized form of the radiative transfer equation within the MOM framework. Section 4 provides a verification of our linearization by detailed comparison with the method of finite differences.

2. The matrix operator method

The matrix operator method or discrete space theory [10,11] allows for an exact and speedy computation of the radiative transfer of turbid media, especially because of its encapsulation of the infinite series of reflections into a single matrix inversion [17]

$$(\mathbf{E}-\mathbf{X})^{-1}-\mathbf{E}=\mathbf{X}+\mathbf{X}^2+\mathbf{X}^3+\cdots, \tag{1}$$

where $\mathbf{X}$ is a matrix representing a pair of consecutive reflections between the two layers and $\mathbf{E}$ is the identity matrix. This eliminates the issue of slow convergence for weakly absorbing atmospheres, i.e., at high values of single scattering albedo, $\overline{\varpi}_0\to 1$, that are faced by several other methods. Also, there is minimal loss of computational speed for increasing optical thicknesses. MOM generates the entire radiative field for a given scenario, both internal and at the boundaries of the atmosphere, for isotropic as well as anisotropic scatterers, and hence can be used for the simultaneous computation of intensities of radiation measured in different viewing geometries. Thus, it lends itself ideally to the simulation of backscattered light arriving at a satellite detector like MISR after reflection through a hazy atmosphere with a dark or reflective underlying surface.

2.1. General formalism

The following equation describes monochromatic, scalar, one-dimensional radiative transfer for an infinitesimal layer in a plane parallel atmosphere:

$$\begin{aligned}\mu\frac{\mathrm{d}L(\overline{\tau},\mu,\phi;\mu_0,\phi_0)}{\mathrm{d}\overline{\tau}} &= -L(\overline{\tau},\mu,\phi;\mu_0,\phi_0)+(1-\overline{\varpi}_0)B(T)\\ &+\frac{\overline{\varpi}_0}{4\pi}\overline{P}(\mu,\phi;\mu_0,\phi_0)S_0\exp(-\overline{\tau}/\mu_0)\\ &+\frac{\overline{\varpi}_0}{4\pi}\int_0^{2\pi}\int_{-1}^{1}\overline{P}(\mu,\phi;\mu',\phi')L(\overline{\tau},\mu',\phi';\mu_0,\phi_0)\,\mathrm{d}\mu'\,\mathrm{d}\phi',\end{aligned} \tag{2}$$

where $\mu_0>0$ is the cosine of the solar zenith angle, ϕ_0 is the solar azimuth angle, S_0 is the incident flux at the top of atmosphere (TOA), $\overline{\tau}$ is the optical depth from the TOA of a layer of average optical thickness $\mathrm{d}\overline{\tau}$, $\overline{\varpi}_0$ is the layer-averaged single scattering albedo, $B(T)$ is Planck's function for temperature T, $\overline{P}$ is the layer-averaged, normalized phase function, and L is the intensity of the diffuse radiation propagating along a direction cosine μ such that $\mu>0$ for the downward and $\mu<0$ for the upward directions. All terms on the right hand side of Eq. (2) having a positive sign denote sources, while the negative sign denotes sinks of radiant energy. Furthermore, the phase function $\overline{P}$ is normalized such that

$$\frac{1}{4\pi}\int_0^{2\pi}\int_{-1}^{1}\overline{P}(\mu,\phi;\mu',\phi')\,\mathrm{d}\mu'\,\mathrm{d}\phi'=1. \tag{3}$$

Expressing the total radiance as the sum of its direct and diffuse parts, so that

$$\begin{aligned}I(\overline{\tau},\mu,\phi;\mu_0,\phi_0) &= S_0\exp(-\overline{\tau}/\mu_0)\delta(\mu-\mu_0)\delta(\phi-\phi_0)\\ &+L(\overline{\tau},\mu,\phi;\mu_0,\phi_0)\end{aligned} \tag{4}$$

we can rewrite Eq. (1) as

$$\begin{aligned}\mu\frac{\mathrm{d}I(\overline{\tau},\mu,\phi;\mu_0,\phi_0)}{\mathrm{d}\overline{\tau}} &= -I(\overline{\tau},\mu,\phi;\mu_0,\phi_0)+(1-\overline{\varpi}_0)B(T)\\ &+\frac{\overline{\varpi}_0}{4\pi}\int_0^{2\pi}\int_{-1}^{1}\overline{P}(\mu,\phi;\mu',\phi')I(\overline{\tau},\mu',\phi';\mu_0,\phi_0)\,\mathrm{d}\mu'\,\mathrm{d}\phi'.\end{aligned} \tag{5}$$

The boundary conditions on the above are

$$I(0,\mu,\phi;\mu_0,\phi_0)=S_0\delta(\mu-\mu_0)\delta(\phi-\phi_0) \tag{6}$$

for $\mu\geq 0$, representing incoming solar radiation at the top of atmosphere, where $\delta(\cdot)$ is the Dirac delta function.

The atmosphere is bounded below by a surface of emissivity $\epsilon(\mu)$ and bidirectional reflectance distribution function $r(\mu,\phi;\mu',\phi')$, so that denoting the total atmospheric optical thickness as $\overline{\tau}_s$ yields

$$\begin{aligned}I(\overline{\tau}_s,\mu,\phi;\mu_0,\phi_0) &= \epsilon(\mu)B(T)\\ &+\int_0^{2\pi}\int_0^{1}r(\mu,\phi;\mu',\phi')I(\overline{\tau}_s,\mu',\phi';\mu_0,\phi_0)\mu'\,\mathrm{d}\mu'\,\mathrm{d}\phi'\end{aligned} \tag{7}$$

with the natural constraint

$$\epsilon(\mu)+\int_0^{2\pi}\int_0^{1}r(\mu,\phi;\mu',\phi')\mu'\,\mathrm{d}\mu'\,\mathrm{d}\phi'=1. \tag{8}$$

Following Chandrasekhar [2], we express the intensity and the phase function (assumed azimuthally symmetric here) using Fourier expansion [5,26,2] as

$$I(\mu,\phi;\mu_0,\phi_0)=\sum_{m=0}^{N}I_m(\mu,\mu_0)\cos m(\phi-\phi_0), \tag{9}$$

$$\overline{P}(\mu,\phi;\mu_0,\phi_0)=\frac{1}{2}\sum_{m=0}^{N}(2-\delta_{0m})\overline{a}_m(\mu,\mu_0)\cos m(\phi-\phi_0), \tag{10}$$

where $N\to\infty$. Substituting Eqs. (9) and (10) in Eq. (5), and writing

$$\overline{P}_m(\mu,\mu')=\left(\frac{2-\delta_{0m}}{2}\right)\overline{a}_m(\mu,\mu') \tag{11}$$

we get

$$\begin{aligned}\mu\frac{\mathrm{d}I_m(\overline{\tau},\mu,\mu_0)}{\mathrm{d}\overline{\tau}} &= -I_m(\overline{\tau},\mu,\mu_0)\\ &+\frac{\overline{\varpi}_0(1+\delta_{0m})}{4}\int_{-1}^{1}\overline{P}_m(\mu,\mu')I_m(\overline{\tau},\mu',\mu_0)\,\mathrm{d}\mu'\\ &+(1-\overline{\varpi}_0)B(T)\delta_{0m},\end{aligned} \tag{12}$$

where $\delta_{ij}=1$ for $i=j$, and $\delta_{ij}=0$ for $i\neq j$. Eqs. (10)–(12) are defined such that, if Eq. (3) holds true, we get for $m=0$

$$\frac{1}{2}\int_{-1}^{1}\overline{P}_0(\mu,\mu')\,\mathrm{d}\mu=1 \tag{13}$$

for all μ'. P_0 represents the azimuthally averaged phase function for scattering between the zenith angles μ' and μ.

Now using a quadrature scheme (Gauss–Lobatto) to discretize the zenith angles, Eq. (12) can be rewritten in

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