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A dynamic multi-scale model for transient radiative transfer calculations

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ABSTRACT

A dynamic multi-scale model which couples the transient radiative transfer equation (RTE) and the diffusion equation (DE) is proposed and validated. It is based on a domain decomposition method where the system is divided into a mesoscopic subdomain, where the RTE is solved, and a macroscopic subdomain where the DE is solved. A buffer zone is introduced between the mesoscopic and the macroscopic subdomains, as proposed by Degond and Jin (2005 [1]), where a coupled system of two equations, one at the mesoscopic and the other at the macroscopic scale, is solved. The DE and the RTE are coupled through the equations inside the buffer zone, instead of being coupled through a geometric interface like in standard domain decomposition methods. One main advantage is that no boundary or interface conditions are needed for the DE. The model is compared to Monte Carlo, finite volume and P1 solutions in one dimensional stationary and transient test cases, and presents promising results in terms of trade-off between accuracy and computational requirements.

1. Introduction

In the last decades, many research fields and applications such as combustion, optical tomography, atmospheric physics or solar energy processes paid more attention to thermal radiation, and various numerical methods have been developed to improve the efficiency of the radiative transfer calculations [2]. Almost all of these methods are based on the radiative transfer equation (RTE) or the diffusion equation (DE). The RTE is a kinetic transport equation for the radiative intensity, defined at the mesoscopic scale, and which represents the most accurate model for thermal radiation. At the mesoscopic scale, the quantities depend on the position, the direction and time, while at the macroscopic scale, the dependence on the direction is not taken into account. The DE is a macroscopic model, where the P1 and the diffusion approximations are assumed, which is much more easier to solve, but remains inaccurate in a lot of physical situations.

Indeed, the P1 approximation requires that the intensity within the medium be nearly isotropic. Calculating the radiative transfer using the DE with the so-called P1 method gives acceptable results in highly scattering media, but displays some major drawbacks in zone where the near-isotropic assumption is not valid, or if the system has subdomain where the medium is not optically thick, or when surface emission dominates over medium emission [2]. Moreover, the definition of the boundary conditions for the DE is a complicated issue where one has to respect constraints which can be very restrictive in numerous applications. Accordingly, the DE fails to describe accurately the light propagation in the proximity of boundaries or sources. Solutions have been proposed to improve the P1 model, such as the high order PN-approximations [3] or the modified and improved differential approximation [4,5], but it is still very complicated to treat the boundary conditions with good accuracy.

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Methods based on the RTE have been implemented and developed over the years such as the finite volume methods (FVMs) [6], the discrete ordinate methods (DOMs) [7], or the Monte Carlo methods (MCMs) [8]. These methods offer a better accuracy than the P1 method [2], but require a higher computational effort due to the high dimensionality of the RTE. In these methods based on the mesoscopic model, the computational requirement is generally increasing with the optical thickness, which is the inverse of the Knudsen number in radiative transfer. Therefore, at large optical thicknesses when the regime is diffusive, a large part of the computational time is due to a part of the system that could be more efficiently described by the DE. In practice, methods such as the FVM or the DOM become inefficient in the diffusive regime since the numerical parameters must respect strong constraints for stability reasons. Typically, in transient calculations, the time step Δt has to be of the order of the square of the mean free path, and the mesh size Δx has to be lower than the mean free path, which is very small in the diffusive regime. Concerning the Monte Carlo method, the number of scattering events to be randomly generated is very large in the diffusive regime and requires consequently an important computational effort. Moreover, convergence difficulties may occur if the Monte Carlo algorithm is not correctly optimized for thick media [9].

Consequently, it seems natural to try to solve each model wherever it is appropriate. In multi-scale problems, where one has to deal with diffusive and kinetic regimes, a solution is to couple the DE with the kinetic model using a domain decomposition method, in which the system is decomposed into a macroscopic and a mesoscopic subdomains. Domain decomposition strategies matching kinetic and diffusion models have received a lot of attention in the last decade [10–13]. In particular, it has been tested in various studies [14–17] in radiative transfer which showed a significant improvement over the P1 method, especially close to the boundaries. However, such strategies have to face the delicate issue of finding the interface conditions at the boundary of two neighboring decomposed domains.

A solution to overcome these difficulties related to the interface treatment is to introduce a buffer zone in which both the macroscopic and the mesoscopic models are solved and coupled, as proposed by Degond and Jin in [1]. In this approach, the two domains overlap but the solution of the original RTE can be recovered as the sum of the solutions of the two equations. The coupling is then applied through the equations rather than the boundary conditions. In the buffer zone, a smooth transition function makes the equations degenerate at the end of the buffer zone, and therefore, no boundary conditions are needed. So, in addition to the cost reduction due to the fact that the RTE is solved only where it is necessary (as in standard domain decomposition approaches), the present transition model overcomes the technical difficulties related to the definition of the boundary conditions of the DE (at the interface with the mesoscopic subdomain or at the frontier of the domain). Note that Degond et al. have also extended this approach for coupling kinetics and hydrodynamics equations in [18–21].

In the present work, a dynamic multi-scale model is developed for transient radiative transfer calculations which couples the RTE with the DE, and is validated on one dimensional test cases. The model is extended in order to be efficient when collimated irradiation is considered at the boundary of the system. The present paper is organized into two main parts: Section 2 presents the dynamic multi-scale model developed for the coupling between the RTE and the DE, and is illustrated in a one dimensional (1D) stationary test case. In Section 3, the model is extended to collimated irradiation and applied to transient radiative transfer. The results are compared with solutions obtained by Monte Carlo, finite volume, and P1 methods.

2. Dynamic multi-scale model for coupling the RTE and the DE

2.1. From the RTE to the DE

The RTE in absorbing and scattering media can be written as

$$\frac{1}{c}\frac{\partial I}{\partial t}+\boldsymbol{u}\cdot\boldsymbol{\nabla} I=\sigma_a I_b-(\sigma_a+\sigma_s)I+\sigma_s\langle pI\rangle, \tag{1}$$

where the mesoscopic unknown $I=I(\boldsymbol{r},\boldsymbol{u},t)$ is the radiative intensity, $\sigma_a=\sigma_a(\boldsymbol{r})$ and $\sigma_s=\sigma_s(\boldsymbol{r})$ are the absorption and scattering coefficients, respectively, $p(\boldsymbol{r},\boldsymbol{u}|\boldsymbol{u}')$ is the normalized scattering phase function depending on the incident direction $\boldsymbol{u}'$ and the scattered one $\boldsymbol{u}$, and $I_b=I_b(\boldsymbol{r},t)$ is the blackbody intensity (all these quantities and coefficients depend on the radiation frequency). For clarity reason, the in-scattering term is denoted by $\sigma_s\langle p(\boldsymbol{r},\boldsymbol{u}|\boldsymbol{u}')I(\boldsymbol{r},\boldsymbol{u}',t)\rangle=\sigma_s\langle pI\rangle$ where the symbol $\langle\cdot\rangle$ represents the integral over the solid-angle space $\langle f\rangle=\int_{4\pi} f\,d\Omega$. The general boundary conditions for an opaque surface with arbitrary surface properties can be written as

$$I_w(\boldsymbol{r}_{\boldsymbol{w}},\boldsymbol{u},t)=\epsilon_w I_b(\boldsymbol{r}_{\boldsymbol{w}},t)+\int_{\boldsymbol{u}\cdot\boldsymbol{n}>0}\rho(\boldsymbol{r}_{\boldsymbol{w}},\boldsymbol{u}|\boldsymbol{u}')I(\boldsymbol{r}_{\boldsymbol{w}},\boldsymbol{u}',t)(\boldsymbol{u}'\cdot\boldsymbol{n})\,d\Omega', \tag{2}$$

where $\boldsymbol{r}_{\boldsymbol{w}}$ is the position at the boundary of the system, $\boldsymbol{n}$ is the local inward surface normal, $\epsilon_w=\epsilon(\boldsymbol{r}_{\boldsymbol{w}},t)$ is the emissivity and ρ is the bidirectional reflection function.

If the RTE is integrated over the solid-angle space 4π, the following equation is obtained:

$$\frac{1}{c}\frac{\partial \Phi}{\partial t}+\boldsymbol{\nabla}\cdot\boldsymbol{q}=\sigma_a[4\pi I_b-\Phi], \tag{3}$$

where Φ is the incident radiation, also called the fluence rate [22], and is defined by $\Phi=\langle I\rangle$. $\boldsymbol{q}(\boldsymbol{r},t)$ is the radiative heat flux defined by $\boldsymbol{q}=\langle I\boldsymbol{u}\rangle$. With the P1 approximation, the radiative intensity is assumed nearly isotropic, and is expressed according to the first order of the spherical harmonic expansion

$$I=\frac{\Phi}{4\pi}+\frac{3}{4\pi}\boldsymbol{q}\cdot\boldsymbol{u}. \tag{4}$$