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Hybrid forward-peaked-scattering-diffusion approximations for light propagation in turbid media with low-scattering regions

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ABSTRACT

Coupled light transport models which use forward-peaked scattering approximations of the radiative transport equation and the diffusion approximation to model light propagation in tissues are introduced. The forward-peaked Fokker–Planck–Eddington approximations are used in those parts of the domain in which the diffusion approximation is not valid, such as close to the source and boundary, and in low-scattering regions. The diffusion approximation is used elsewhere. The models are coupled through boundary conditions and the resulting system of equations is solved using a finite element method. The proposed coupled Fokker–Planck-diffusion and Fokker–Planck–Eddington-diffusion models are tested with simulations, and compared with the radiative transport equation, diffusion approximation and coupled radiative transport-diffusion model. The results show that the new coupled models give almost as accurate results as the radiative transport equation with reduced computational load.

1. Introduction

Non-invasive optical imaging modalities provide useful information about tissue health. By measuring light propagation through tissues one can derive images of the optical properties of the medium. These imaging modalities include, for example, diffuse optical tomography (DOT) in which the optical properties of tissues are reconstructed on the basis of the transmission measurements of scattered near-infrared light on the surface of the object. Medical applications of DOT include breast cancer detection, monitoring of infant brain oxygenation level and functional brain activation studies, for reviews see e.g. [1–5].

Image reconstruction problem in DOT is a non-linear ill-posed inverse problem. Thus, even small errors in measurements or modelling can cause large errors in the reconstructed images. Moreover, the iterative solution of the non-linear problem requires several solutions of the corresponding forward model. Therefore, an accurate and computationally feasible forward model is needed.

Light propagation in tissues is governed by the radiative transport equation (RTE) [1,4,6,7]. The RTE takes into account absorption and multiple scattering due to inhomogeneities in the medium. The RTE does not have an analytical solution in arbitrary geometry and numerical methods are computationally expensive due to a large number of variables in the RTE. Therefore, the RTE is often approximated by some computationally less demanding model.

The most often used approximation to the RTE is the diffusion approximation (DA). In the DA, one assumes that light becomes almost isotropic due to strong scattering. In addition, scattering must be much stronger than absorption. Due to these limitations, the DA fails to describe light propagation accurately close to the boundary, sources, and in low-scattering and non-scattering regions [1,8–10]. A typical low-scattering region encountered in optical imaging

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of the brain is the cerebrospinal fluid around the brain and in the ventricles.

To overcome the limitations of the diffusion approximation, different hybrid models have been introduced. The hybrid models include the radiosity diffusion model [10,11], hybrid Monte Carlo-diffusion models [9,12–14], hybrid radiative transport-diffusion models [15–18], and the variable order spherical harmonics (P_n) approximation to the RTE [19,20]. The radiosity diffusion model combines the diffusion theory with a ray-tracing algorithm and is applicable in highly scattering medium with non-scattering regions. The hybrid Monte Carlo-diffusion models can be used in complex heterogeneous media but suffer from a time-consuming nature of Monte Carlo simulations and often require an iterative mapping between the models in order to take into account back-scattering between the sub-domains leading to computationally excessive problems. In the hybrid radiative transport-diffusion models, light propagation is modeled using the RTE in sub-domains in which the DA is not valid. The DA is used in the rest of the domain and the models are coupled with boundary conditions at the interfaces between the sub-domains . The variable order spherical harmonics approximation uses different orders of the P_n-approximation in each sub-domain depending on local scattering and absorption properties.

In tissues, scattering is typically forward-peaked, indicating that the direction of photons changes only a little in scattering events. The numerical solution of the RTE with forward-peaked scattering is challenging due to dense angular discretization needed to describe scattering accurately. Motivated by that, Fokker–Planck–Eddington-approximations have been proposed to approximate the RTE [21–29]. These approximations include, for example, the Fokker–Planck (FP) and Fokker–Planck–Eddington (FPE) equations. These approximations take into account forward-peaked and large-angle scattering analytically by approximating the scattering probability distribution using a linear combination of delta functions and Legendre polynomials. As a result, forward-peaked scattering probability distribution is replaced by an angular differential operator together with a smooth integral operator. Thus, coarser angular discretization can be used compared with the RTE leading to computational savings [24,27].

In this paper, we use the Fokker–Planck–Eddington-approximations to approximate the RTE. We introduce coupled Fokker–Planck-diffusion (cFP-DA) and coupled Fokker–Planck–Eddington-diffusion (cFPE-DA) models for modelling light propagation in tissues. The coupled models are solved with a finite element method (FEM). We compare the results of the coupled models with the solutions of the RTE, FP, FPE, and DA, and with the previously developed [17] coupled RTE-DA (cRTE-DA).

The rest of the paper is organized as follows. The light transport models including the forward-peaked scattering approximations are reviewed and the coupled models are introduced in Section 2. The numerical solutions of the coupled models using the FEM are described in Section 3. In Section 4 we test the proposed coupled models with simulations. Section 5 gives the conclusions.

2. Light transport models

2.1. The radiative transport equation

A widely accepted model for light propagation in tissues is the radiative transport equation [7]. The RTE is a one-speed approximation of the Boltzmann transport equation, and thus energy is assumed to be preserved in scattering events. In addition, the refractive index is assumed to be constant within the medium. The RTE neglects wave phenomena such as diffraction and interference, and treats photons as particles which propagate along straight lines between scattering and absorption events.

Let $\Omega \subset \mathbb{R}^n$ be the physical domain, and $n=2,3$ be the dimension of the domain. In addition, let $\hat{\mathbf{s}} \in \mathbb{S}^{n-1}$ denote a unit vector in the direction of interest on the unit sphere $\mathbb{S}^{n-1}$. The frequency domain version of the RTE without internal sources is

$$\frac{\mathrm{i}\omega}{c}\phi(\mathbf{r},\hat{\mathbf{s}})+\hat{\mathbf{s}}\cdot\nabla\phi(\mathbf{r},\hat{\mathbf{s}})+\mu_{\mathrm{a}}\phi(\mathbf{r},\hat{\mathbf{s}})=\mu_{\mathrm{s}}\mathcal{L}\phi(\mathbf{r},\hat{\mathbf{s}}), \tag{1}$$

where i is the imaginary unit, ω is the angular modulation frequency of the input signal (the units of Hz), c is the speed of light in the medium (the units of m/s), $\phi(\mathbf{r},\hat{\mathbf{s}})$ is the radiance (the units of $\mathrm{W\,m^{-2}\,sr^{-1}}$ in 3D and $\mathrm{W\,m^{-1}\,rad^{-1}}$ in 2D), and $\mu_{\mathrm{s}}=\mu_{\mathrm{s}}(\mathbf{r})$ and $\mu_{\mathrm{a}}=\mu_{\mathrm{a}}(\mathbf{r})$ are the scattering and absorption coefficients of the medium (the units of $\mathrm{m^{-1}}$), respectively [1,30,31].

The scattering operator $\mathcal{L}_{\mathrm{RTE}}$ is

$$\begin{aligned}\mathcal{L}\phi(\mathbf{r},\hat{\mathbf{s}}) &= \mathcal{L}_{\mathrm{RTE}}\phi(\mathbf{r},\hat{\mathbf{s}})\\ &= -\phi(\mathbf{r},\hat{\mathbf{s}})+\int_{\mathbb{S}^{n-1}}\Theta(\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')\phi(r,\hat{\mathbf{s}}')\,\mathrm{d}\hat{\mathbf{s}}',\end{aligned} \tag{2}$$

where the scattering phase function $\Theta(\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')$ describes the probability for a photon to scatter from direction $\hat{\mathbf{s}}'$ in direction $\hat{\mathbf{s}}$. An often used phase function is the Henyey–Greenstein scattering function [32]

$$\Theta(\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')=\frac{1}{|\mathbb{S}^{n-1}|}\frac{1-g^2}{(1+g^2-2\,g\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')^{n/2}}, \tag{3}$$

where scattering shape parameter g defines the shape of the probability distribution, and $|\mathbb{S}^{n-1}|$ is the surface measure of $\mathbb{S}^{n-1}$ ($|\mathbb{S}^1|=2\pi$ and $|\mathbb{S}^2|=4\pi$). The angular moments of the scattering phase function are

$$g_m=\int_{\mathbb{S}^{n-1}}(\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')^m\Theta(\hat{\mathbf{s}}\cdot\hat{\mathbf{s}}')\,\mathrm{d}\hat{\mathbf{s}}. \tag{4}$$

In the case of the Henyey–Greenstein phase function angular moments are $g_m=g^m$. In biological tissues, the first moment is typically close to 1 indicating that scattering is forward-peaked [33]. Thus, the direction of photons is most likely to change only a little in scattering events.

In this paper, it is assumed that no photons travel in an inward direction at the boundary $\partial\Omega$ except at the source location $\varepsilon_t \subset \partial\Omega$

$$\phi(\mathbf{r},\hat{\mathbf{s}})=\begin{cases}\phi_0(\mathbf{r},\hat{\mathbf{s}}), & \mathbf{r}\in\varepsilon_t, \quad \hat{\mathbf{s}}\cdot\hat{\mathbf{n}}<0,\\ 0, & \mathbf{r}\in\partial\Omega\backslash\varepsilon_t, \quad \hat{\mathbf{s}}\cdot\hat{\mathbf{n}}<0,\end{cases} \tag{5}$$