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Determining energy flow propagation direction of transmitted wave at an active medium–vacuum interface

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ABSTRACT

By simultaneously considering the real valued boundary conditions and Poynting theorem, time dependent Poynting flows of reflected and transmitted waves at an active medium-vacuum interface are determined uniquely. Then propagation direction of the transmitted wave is given according to its time averaged Poynting flow. Numerical simulations demonstrate that, at a high gain or loss active medium–vacuum interface, significant difference between electric and magnetic damping angle may induce energy flow propagation direction of the transmitted wave to deviate strongly away from that obtained by the usual Snell's law and to arise negative refraction generally. Our work provides a convenient way to address problems of reflection and refraction at an active media–vacuum interface.

1. Introduction

Experimental verification [1,2] of negative refraction of the waves at an interface formed by left-handed material (LHM) and usual right-handed material (RHM) arouse great interest in the unusual electromagnetic properties of LHM [3–21]. Recently, the active media have been suggested to either overcome the loss problem of the passive LHM or to produce isotropic negative-index media at optical frequencies [5–14]. Usual studies mainly focused on either the handedness or sign of refraction index of the materials [5–14]. A variety of conditions for left handedness and negative refraction index have been proposed in the literatures [15,16]. On the other hand, previous studies have been shown that, due to existence of energy loss (the gain may be taken as negative loss), the negative refraction may occur without negative refraction index, whereas, the negative refraction index does not ensure arising the negative refraction [20,21], which is mainly attributed to the fact that the usual Snell's law is no longer valid at a lossy interface [22]. Therefore, investigating effects of energy gain or loss on energy flow propagation direction of transmitted wave are necessary to fully understand the refraction behavior of waves at an active medium–vacuum interface.

The electric and magnetic response properties of homogeneous isotropic active medium are usually described by the complex valued scalar permittivity $\tilde{\varepsilon}=|\tilde{\varepsilon}|\exp(j\alpha_\varepsilon)$ (in this paper, the complex valued parameters are marked with "$\sim$") and permeability $\tilde{\mu}=|\tilde{\mu}|\exp(j\alpha_\mu)$ with $\alpha_{\varepsilon(\text{or}\mu)}\in(\pi,2\pi)$ (e.g., for two-component system) or $\alpha_{\varepsilon(\text{or}\mu)}\in(-\pi,0)$ (for inverted system), respectively [5]. Apparently, evolution of electric field intensity $\vec{E}(t)$, electric displacement vector $\vec{D}(t)$, magnetic field intensity $\vec{H}(t)$ and magnetic flux density $\vec{B}(t)$ of electromagnetic wave in an active medium, as like as in a lossy passive medium, and then around their interface, is often out of step, which differs significantly with the cases of electromagnetic wave in the lossless media.

At a lossy interface, phase-matching condition and complex valued boundary conditions require the normal components of $\tilde{D}_n(t)$ and $\tilde{B}_n(t)$ and the tangential components of $\tilde{E}_\tau(t)$ and $\tilde{H}_\tau(t)$ in both sides of the boundary to be

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equal to each other and to evolve in step, respectively, which may be satisfied only under a certain conditions (e.g., the corresponding waves are taken as harmonic inhomogeneous plane waves (HIPWs)) [21]. Usually, HIPWs instead of the well known harmonic homogeneous plane waves (HHPWs) are adopted to derive the generalized laws of reflection and refraction [22]. The energy flows and energy balance of waves at the interface is discussed by combining Poynting theorem, which is more difficult and complicated than that at a lossless interface [23]. Generally, a deflection angle between planes of incidence and refraction has to be introduced [22]. To the best of our knowledge, experiment verification of the deflection angle between planes of incidence and refraction has never been reported. On the other hand, the real valued boundary conditions are demonstrated to be valid for both HHPWs and HIPWs [21]. The real valued boundary conditions require the parameters of $D_n(t) \equiv \mathrm{Re}(\tilde{D}_n(t))$, $B_n(t) \equiv \mathrm{Re}(\tilde{B}_n(t))$, $E_\tau(t) \equiv \mathrm{Re}(\tilde{E}_\tau(t))$ and $H_\tau(t) \equiv \mathrm{Re}\ (\tilde{H}_\tau(t))$ in both sides of the boundary to be equal to each other at any moment but do not require parameters of $\tilde{D}_n(t)$, $\tilde{B}_n(t)$, $\tilde{E}_\tau(t)$ and $\tilde{H}_\tau(t)$ to evolve in step, respectively. Thus, based on the real valued boundary conditions, HHPWs instead of HIPWs may also be adopted to address the problems of reflection and refraction at a lossy interface. Since the complicated electromagnetic wave may be linearly composited by either HHPWs or HIPWs, the obtained formulas based on HHPWs do not lose any generality.

In this work, we shall take a time domain method to conveniently address problems of reflection and refraction associated with non-synchronous evolution of $\vec{E}(t)$, $\vec{D}(t)$, $\vec{H}(t)$ and $\vec{B}(t)$ at an active medium–vacuum interface [21]. The remainder of the paper is organized as follows: In Section 2, the formulas of reflection and refraction are obtained by simultaneously considering the real valued boundary conditions and Poynting theorem. In Section 3, several novel properties of transmitted wave induced by media gains or losses are addressed. Finally, some conclusions are shown in Section 4.

2. Derivation of the formulas of reflection and refraction

For simplicity, we consider in detail the case of an obliquely incident transverse electric (TE) polarized HHPW traveling from homogeneous isotropic medium 1 into vacuum. Since evolution of $\vec{E}(t)$, $\vec{D}(t)$, $\vec{H}(t)$ and $\vec{B}(t)$ of the wave in a gain or lossy medium are out of step, there are eight possible direction relations among $\vec{E}_i(t)$, $\vec{B}_i(t)$ and $\vec{H}_i(t)$. Based on the real valued boundary conditions and Poynting theorem, the eight direction relations among $\vec{E}_i(t)$, $\vec{B}_i(t)$ and $\vec{H}_i(t)$, and corresponding directions of other parameters (including time dependent Poynting flows (TDPFs), $\vec{E}_r(t)$, $\vec{B}_r(t)$, $\vec{H}_r(t)$, $\vec{E}_t(t)$, $\vec{B}_t(t)$ and $\vec{H}_t(t)$) are given in Fig. 1(a)–(h), respectively. Apparently, here, the electric field intensity $\vec{E}_t(t)$ of transmitted wave evolves in step with $\vec{E}_i(t)$ of the incident wave.

Set the directions of $\hat{e}_E^\varsigma$ for electric field intensity, $\hat{e}_B^\varsigma$ for magnetic flux density, $\hat{e}_H^\varsigma$ for magnetic field intensity and $\hat{e}_S^\varsigma$ for TDPF presented in Fig. 1(a) as the positive directions of each parameter ($\hat{e}_E^\varsigma$, $\hat{e}_B^\varsigma$, $\widehat{e}_H^\varsigma$ and $\hat{e}_S^\varsigma$ are unit vectors, $\varsigma = i,r,t$ refers the incident, reflected and transmitted wave), the signs of the parameters are subsequently taken as to be positive, otherwise, the sign becomes to be negative. The electric field intensity of the TE polarized HHPW is given as $\tilde{\vec{E}}_\varsigma(\vec{r},t) = \tilde{E}_{\varsigma 0}\exp(-j\omega t + j\tilde{\vec{k}}_\xi \cdot \vec{r}_\varsigma)\widehat{e}_E^\varsigma$. Where, ω is a given angular frequency, $\tilde{k}_\xi = (\omega/c)\sqrt{\tilde{\mu}_\xi \tilde{\varepsilon}_\xi}$ propagation constant with $\alpha_{k_\xi} = \alpha_{\mu_\xi} + \alpha_{\varepsilon_\xi}/2$, and $\xi = 1,2$ refers the medium 1 and vacuum, respectively. The corresponding magnetic field intensity and magnetic flux density may be obtained as $\tilde{\vec{H}}_\varsigma(\vec{r},t) = (\tilde{E}_\varsigma(\vec{r},t)/\tilde{\eta}_\xi)\hat{e}_H^\varsigma$ ($\tilde{\eta}_\xi = \sqrt{\tilde{\mu}_\xi/\tilde{\varepsilon}_\xi}$ is wave impedance with $\alpha_{\eta_\xi} = \alpha_{\mu_\xi} - \alpha_{\varepsilon_\xi}/2$) and $\tilde{\vec{B}}_\varsigma(\vec{r},t) = \tilde{k}_\xi \tilde{E}_\varsigma(\vec{r},t)\hat{e}_B^\varsigma$, respectively. At the interface, the real valued parameters may be written as $\vec{E}_\varsigma(t) \equiv \mathrm{Re}(\tilde{\vec{E}}_\varsigma(t))$, $\vec{H}_\varsigma(t) \equiv a_\varsigma \mathrm{Re}(\tilde{E}_\varsigma(t))\hat{e}_H^\varsigma$ (with $a_\varsigma = [\mathrm{Re}(\tilde{\eta}_\xi) + \mathrm{Im}(\tilde{\eta}_\xi)\mathrm{Im}(\tilde{E}_\varsigma)/\mathrm{Re}(\tilde{E}_\varsigma)]/\tilde{\eta}\tilde{\eta}^*$), and $\vec{B}_\varsigma(t) \equiv b_\varsigma \mathrm{Re}(\tilde{E}_\varsigma(t))\hat{e}_B^\varsigma$ (with $b_\varsigma = \mathrm{Re}(\tilde{k}_\xi) - \mathrm{Im}(\tilde{k}_\xi)\mathrm{Im}(\tilde{E}_\varsigma)/\mathrm{Re}(\tilde{E}_\varsigma)$). Thus the real valued boundary conditions are taken as following equations of (1)–(3).

$$\mathrm{Re}(\tilde{E}_i(t)) + \mathrm{Re}(\tilde{E}_r(t)) = \mathrm{Re}(\tilde{E}_t(t)), \tag{1}$$

$$a_i \mathrm{Re}(\tilde{E}_i(t))\cos\theta_i - a_r \mathrm{Re}(\tilde{E}_r(t))\cos\theta_r = a_t \mathrm{Re}(\tilde{E}_t(t))\cos\theta_t, \tag{2}$$

$$b_i \mathrm{Re}(\tilde{E}_i(t))\sin\theta_i + b_r \mathrm{Re}(\tilde{E}_r(t))\sin\theta_r = b_t \mathrm{Re}(\tilde{E}_t(t))\sin\theta_t. \tag{3}$$

It is stressed that, here, to treat non-synchronized evolution of $\tilde{E}(t)$, $\tilde{B}(t)$ and $\tilde{H}(t)$ at the gain or lossy interface, the time term is remained in the boundary conditions.

According to Eqs. (1) and (3), we have

$$b_i \sin\theta_i = b_r \sin\theta_r = b_t \sin\theta_t. \tag{4}$$

For lossless media, $b_\varsigma = k_\xi$, Eq. (4) is just the phase-matching condition. From Eq. (4) the generalized Snell's law including effects of energy losses is obtained as

$$\theta_r = \theta_i, \tag{5a}$$

$$\sin\theta_t = b_i \sin\theta_i / b_t = \sin\theta_i \frac{\mathrm{Re}(\tilde{k}_1)}{\mathrm{Re}(\tilde{k}_2)} \frac{1 + tg(\alpha_{k_1})tg(\omega t)}{1 + tg(\alpha_{k_2})tg(\omega t)}. \tag{5b}$$

Further, solving Eqs. (1) and (2) gives the formula for transmission and reflection coefficients

$$T_E(t) \equiv \frac{\mathrm{Re}(\tilde{E}_t(t))}{\mathrm{Re}(\tilde{E}_i(t))} = \frac{a_r\cos\theta_r + a_i\cos\theta_i}{a_t\cos\theta_t + a_r\cos\theta_r}, \tag{6a}$$

$$\Gamma_E(t) \equiv \frac{\mathrm{Re}(\tilde{E}_r(t))}{\mathrm{Re}(\tilde{E}_i(t))} = \frac{a_i\cos\theta_i - a_t\cos\theta_t}{a_t\cos\theta_t + a_r\cos\theta_r}. \tag{6b}$$

According to TDPF expression of $\vec{S}(t) \equiv \vec{E}(t) \times \vec{H}(t)$, at a certain time range, TDPF of incident wave may be smaller than zero due to non-synchronous evolution of $\tilde{E}_i$ and $\tilde{H}_i$, i.e., the time dependent energy flow of incident wave propagates away from the interface (see Fig. 1(b), (e), (g) and (h), respectively.). Contrarily, TDPF of the usually termed "refracted wave" propagates toward the interface, i.e. the "refracted wave" plays a role as "incident one". Thus the parameters of a_i and a_r in Eq. (2) and (6) become to be $|a_i|$ and $|a_r|$, respectively, which satisfy causality and the law of energy conservation. In addition, we see