Journal of Quantitative Spectroscopy & Radiative Transfer 117 (2013) 133–139

Contents lists available at SciVerse ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Coherent backscattering enhancement in medium with variable refractive index

Ya.A. Ilyushin [a,b,*]

[a] Physical Faculty, Moscow State University, 119992 GSP-2, Lengory, Moscow, Russia
[b] Kotel'nikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences Mokhovaya st., 11-7, Moscow 125009, Russia

ARTICLE INFO

Article history:
Received 6 September 2012
Received in revised form
12 December 2012
Accepted 14 December 2012
Available online 25 December 2012

Keywords:
Radiative transfer
Coherent backscattering enhancement
Weak localization
Refractive index

ABSTRACT

The paper addresses the problem of coherent backscattering enhancement in the refracting medium. Formulation of the problem on the basis of the radiative transfer theory for refracting media is derived. Results of numerical simulations for conservative Rayleigh slab medium reveal the moderate impact of the refraction on the shape of the coherent backscattering peak. For all the basic fundamental types of radiation sources (flat directed, point isotropic, point directed) asymptotic solutions of the radiative transfer equation in refracting medium are derived in the small angle approximation. Approximate solution for the wings of the coherent backscattering peak is presented.

1. Introduction

Weak localization of waves (coherent backscattering enhancement) is extensively studied since 80th years of the last century [1,2]. Since that, a large number of papers on the subject, both theoretical and experimental, have been published, so the phenomenon has been investigated in many details. Many different approaches have been applied for theoretical description of the weak localization, including the simplest ones like double scattering approximation [1], diffusion approximation [2], and more sophisticated analytical and numerical techniques of the radiative transfer [3–5] and electromagnetic theory [6]. The role of the polarization state of the radiation [7–9], confined geometry of the medium [10] etc. has been revealed and estimated.

However, vast majority of these research have been focused on the macroscopically uniform and homogeneous media. Role of the refraction in the medium, although it is well known in general radiative transfer theory and therefore accounted for, e.g. in plasma radiative transport [11], seems to be almost neglected in weak localization studies. Coherent backscattering (CB) from the medium with nonunity but constant refraction index was investigated in [12]. Weak localization in media with the graded refractive index has not yet been studied, to the authors knowledge.

Nevertheless, a combination of random scattering and regular refractive index gradients is often encountered in artificial and natural media. A number of technological objects, where graded refractive index impacts the radiative heat transfer, are discussed in [13]. In addition, objects of that type are dealt with in radar probing, where the coherent backscattering effects were first revealed more than 20 years ago [14]. Ionospheric and solar coronal plasma, where turbulent and other fluctuations of the plasma density are superimposed onto its layered stratification [15,16], and terrestrial and planetary snow and ice covers [17,18], show examples of radar targets of this type. Motivation of this study is the need in assessment of the impact of the graded refractive index on the coherent backscattering enhancement.

* Correspondence address: Physical Faculty, Moscow State University, 119992 GSP-2, Lengory, Moscow, Russia. Tel.: +7 495 939 3252.
E-mail addresses: ilyushin@physics.msu.ru, rx3ahl@mail.ru

0022-4073/$ - see front matter
http://dx.doi.org/10.1016/j.jqsrt.2012.12.006

Nomenclature

C_j	coefficients of the spherical harmonic expansions
F	source term in RTE
g	scattering anisotropy parameter
k_y	Fourier transform parameter
k	wave number
l	mean free path length
L	angular and spatial intensity distribution
$L_0(\cdot)$	unscattered part of the radiation
$\tilde{L}$	Fourier transform of the radiance
$\tilde{L}_b$	Fourier transform of the backscattered radiation
L_D	diffuse part of the RTE solution
L_a	anisotropic part of the RTE solution
n	refractive index
$\mathbf{n}_\perp$	transverse direction vector
$P_j(\cdot)$	Legendre polynomials
r	radius
$\mathbf{r}$	radius vector
$\mathbf{p}$, $\mathbf{q}$	vector parameters of the Fourier transform
$\mathbf{r}_\perp$	transverse radius vector
S	scattering term in RTE
s	optical path
x,y,z	Cartesian coordinates
$x(\mathbf{\Omega},\mathbf{\Omega}')$	phase scattering function
$Y_j^m(\mathbf{\Omega})$	spherical harmonics
$\mathbf{\Omega}$	unit vector of direction
ε	volume extinction coefficient
γ	logarithmic derivative of the refractive index
$\delta(\cdot)$	Dirac delta function
μ,μ_x,μ_y,μ_z	directional cosines
Λ	single scattering albedo
τ_0	total optical thickness of the slab
$\chi(\cdot)$	Hankel transform of the scattering phase function
σ	scattering cross section
σ_b	backscattering differential cross section
θ	phase angle
Θ	direction polar angle
ξ,η,ζ	characteristic variables

2. Theory

It is well known [1] that the so-called cyclic scattering diagrams are responsible for the weak localization in scattering medium. Due to equivalence between cyclic and ladder diagrams arising from the reciprocity principle [7,8], the problem is reduced to the radiative transfer problem for the point directed (PD) source [5]

$$L_0(\mathbf{r},\mathbf{\Omega}) = \delta(x)\delta(y)\delta(\mathbf{\Omega}) \exp\left(-\int_0^z \varepsilon\, dz\right). \tag{1}$$

According to these papers, intensity of the coherently scattered radiation is proportional to the Fourier component of the corresponding solution of the radiative transfer equation (RTE) with the Fourier parameter $k_y = k\theta$, where θ is the phase angle.

The scalar radiative transfer equation in refractive medium [11]

$$n^2 \frac{d}{ds}\left(\frac{L}{n^2}\right) + \varepsilon L = S + F, \tag{2}$$

where S and F are the scattering and source terms, respectively, n is the refractive index of the medium. To our knowledge, Woolley was the first to derive Eq. (2) (see the reference in [19]). Since that, several other equations of that type have been derived [20–23]. Nevertheless, Eq. (2) is assumed in most investigations. A special form of RTE for the plane parallel layered stratified medium has been derived in [24]. In the paper [25], the RTE in cylindrical and spherical coordinate system for arbitrary refractive index distribution may be found.

Make use of the RTE for the flat layered refracting medium [24]

$$(\mathbf{\Omega}\cdot\nabla)L + \gamma(1-\mu_z^2)\frac{dL}{d\mu_z} = -\varepsilon L + \mu_z 2\gamma L$$
$$+ \frac{\Lambda\varepsilon}{4\pi}\oint L(\mathbf{r},\mathbf{\Omega}')x(\mathbf{\Omega},\mathbf{\Omega}')\, d\mathbf{\Omega}' + f(\mathbf{r},\mathbf{\Omega}), \tag{3}$$

where $\mathbf{\Omega} = (\mu_x,\mu_y,\mu_z)$ is the unit vector of the direction, $\mathbf{r} = (x,y,z)$, $x(\mathbf{\Omega},\mathbf{\Omega}')$ is the phase scattering function, $L(\mathbf{r},\mathbf{\Omega})$ is the angular distribution of the radiance, Λ is the single scattering albedo, ε is the volume extinction coefficient, $\gamma = d\ln n/dz$. Thus, we derive from (3) the equation for the Fourier component $\tilde{L}(z,\mathbf{\Omega})$ [5]

$$\mu_z\frac{d\tilde{L}}{dz} - ik_y\mu_y\tilde{L} + \gamma(1-\mu_z^2)\frac{d\tilde{L}}{d\mu_z} = -\varepsilon\tilde{L} + \mu_z 2\gamma\tilde{L}$$
$$+ \frac{\Lambda\varepsilon}{4\pi}\oint \tilde{L}(z,\mathbf{\Omega}')x(\mathbf{\Omega},\mathbf{\Omega}')\, d\mathbf{\Omega}' + \tilde{f}(z,\mathbf{\Omega}), \tag{4}$$

where the source function is

$$\tilde{f}(z,\mathbf{\Omega}) = \int \exp(ik_y y)\frac{\Lambda}{4\pi}\oint L_0(\mathbf{r},\mathbf{\Omega}')x(\mathbf{\Omega},\mathbf{\Omega}')\, d\mathbf{\Omega}'\, dx\, dy$$
$$= \frac{\Lambda}{4\pi}x(\mathbf{\Omega}) \exp\left(-\int \varepsilon\, dz\right). \tag{5}$$

Equations for the singly scattered intensity, which does not contribute to the coherent backscattering and should be subtracted from the whole RTE solution [7], can be obtained from (3), (4) letting there $\Lambda = 0$. In this paper we restrict our attention to the case $\gamma = const$, i.e. $n(z) = \exp(\gamma z)$.

3. Numerical results

In the present study, we restrict our consideration to the Rayleigh scattering within the scalar radiative transfer theory. In this case there is no need in the separation of the anisotropic components, and Eq. (4) can be solved immediately, e.g. with the finite difference techniques. Here the discrete ordinate (DO) method and upwind-difference scheme [26] has been applied, the scattering integral was evaluated with the spherical quadrature