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Modifications of discrete ordinate method for computations with high scattering anisotropy: Comparative analysis

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ABSTRACT

A numerical accuracy analysis of the radiative transfer equation (RTE) solution based on separation of the diffuse light field into anisotropic and smooth parts is presented. The analysis uses three different algorithms based on the discrete ordinate method (DOM). Two methods, DOMAS and DOM2+, that do not use the truncation of the phase function, are compared against the TMS-method. DOMAS and DOM2+ use the Small-Angle Modification of RTE and the single scattering term, respectively, as anisotropic parts. The TMS method uses the Delta-M method for truncation of the phase function along with the single scattering correction. For reference, a standard discrete ordinate method, DOM, is also included in analysis. The obtained results for cases with high scattering anisotropy show that at low number of streams (16, 32) only DOMAS provides an accurate solution in the aureole area. Outside aureole, the convergence and accuracy of DOMAS, and TMS is found to be approximately similar: DOMAS was found more accurate in cases with coarse aerosol and liquid water cloud models, except low optical depth, while the TMS showed better results in case of ice cloud.

1. Introduction

This paper continues analysis of the scalar radiative transfer equation (RTE) with highly asymmetric phase function in the framework of the discrete ordinates method (DOM) [4,21]. In our recent paper [12], a particular attention was paid to the methods based on decomposition of the diffused light field into a smooth (regular) part and analytically expressed anisotropic part without truncation of the phase function. With anisotropy subtraction, the regular part of the signal, which requires a numerical solution, is essentially smoothed as a function of angles.

In DOM, the view zenith angle (VZA) anisotropy of the signal is expressed via an even number $2N$ of linear differential equations in the system. Each ordinate corresponds to one equation, and there are N ordinates per hemisphere. The azimuthal dependence of radiance is expressed via the Fourier series with M harmonics, where the system of N linear equations is solved independently for each $m=0,\ldots,M$ [23] providing solution in $i=1,\ldots,N$ discrete points $-1<\mu_i<+1$; $\mu_i\neq 0, \pm 1$.

Our previous work [12] showed that anisotropy subtraction using a Small-Angle Modification of RTE, implemented in code DOMAS, accelerated azimuthal convergence of solution significantly, by a factor of three. However, contrary to our expectations, this method did not improve convergence in zenith angle, meaning that a large number of streams would still be required for high accuracy computations with very asymmetric phase functions. It is worth mentioning that accuracy comparison for

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different number of streams N in [12] used cubic spline interpolation to yield solution at selected angles. This method was criticized by Karp [10] as limiting the computational accuracy. A convenient form of computation for an arbitrary angle using integration of the source function was introduced in DOM by Kourganoff [13]. The current work employs the idea of "natural" interpolation by including the required view angles as dummy nodes $-1 \le \mu_d \le +1$ into DOM scheme with zero weighting coefficients $w_d = 0$ [3]. This new approach yields the high accuracy solution with low number of streams. Below, we provide code details and a comparison with other approaches for three cases with high scattering anisotropy, including coarse aerosol fraction and liquid water and ice cloud models.

This paper is structured as follows: Section 2 defines the problem and describes the main characteristics of the methods compared in the paper. The definition of the scenarios for numerical tests is given in Section 3 followed by discussion of the results in Section 4. The paper is concluded with the summary.

2. Definition of the problem

For simplicity, we consider the boundary problem for the scalar RTE and plane-parallel homogeneous atmosphere illuminated at the right angle [4]

$$\begin{cases} \mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega_0}{2}\int_{-1}^{1} p(\mu,\mu')I(\tau,\mu')d\mu' + \frac{\omega_0}{2}p(\mu,1)\exp(-\tau), \\ I(\tau,\mu^+) = 0; \;\; I(\tau_0,\mu^-) = 0. \end{cases} \quad (1)$$

Here, $I(\tau,\mu)$ is the radiance given as a function of optical depth τ $(0 \le \tau \le \tau_0)$ and a cosine of VZA $\mu = \cos\theta$. The surface is assumed to be black. The media scattering properties are given by the single scattering albedo (SSA) ω_0 and phase function $p(\mu,\mu')$. The z-axis is pointed downwards, so that $0^\circ \le \theta < 90^\circ$ $(\mu^+ > 0)$ and $90^\circ < \theta \le 180^\circ (\mu^- < 0)$ correspond to transmitted and reflected radiation, respectively.

The phase function is expanded in the Legendre series

$$p(\mu,\mu') = \sum_{k=0}^{K_{max}} (2k+1)x_k P_k(\mu)P_k(\mu'), \quad (2)$$

where $P_k(\mu)$ is the Legendre polynomial of degree k, x_k are expansion moments, and K_{max} is the maximum expansion order necessary for accurate representation of the phase function which will be denoted hereafter as K if the number of terms involved is less than K_{max}.

The discrete ordinate method is often used to solve Eq. (1). Using the double-Gauss quadrature [22], the scattering integral in Eq. (1) is expressed as a sum in the form

$$\int_{-1}^{1} p(\mu,\mu')I(\tau,\mu')d\mu' \approx \sum_{j=1}^{N} w_j p(\mu,\mu_j^-)I(\tau,\mu_j^-) + \sum_{j=1}^{N} w_j p(\mu,\mu_j^+)I(\tau,\mu_j^+), \quad (3)$$

where w_j are the weighting coefficients, μ_j are the nodes (zeros) of the Legendre polynomial $P_N(\mu)$. Eq. (3) yields the system of $2N$ linear differential equations for Eq. (1). While parameters K in Eq. (2) and N in Eq. (3) seem to be independent, it was shown that $N = K/2$ gives numerically stable results [23]. Thus $N = K/2$ is assumed in Section 2.

The right-hand side of the RTE Eq. (1) is called the source function [4]. The free term of the source function contains all K_{max} moments of the phase function

$$\frac{\omega_0}{2}p(\mu,1)\exp(-\tau) = \frac{\omega_0}{2}\exp(-\tau)\sum_{k=0}^{K_{max}}(2k+1)x_k P_k(\mu)P_k(1), \quad (4)$$

regardless of the number of moments K of the phase function under the scattering integral. The acronym DOM will be used further in this paper for the traditional discrete ordinate method defined by Eqs. (1)–(4) without any modifications. Note that for the azimuthally independent case the single scattered radiation is included in DOM exactly.

Large particles as in clouds, snow, coarse aerosol fraction etc. cause a strong forward scattering and peaks in the backscattering directions. In these cases, K-parameter in Eq. (2) is large, $\sim 10^3$ as well as the number $2N$ of DOM equations. At large N, the matrix of the system easily becomes ill-conditioned, and its solution is time consuming.

Presently, there are two main approaches to solve the RTE problem with high scattering anisotropy. The first one uses different truncation approximations of phase function. These methods were recently analyzed by Rozanov and Lyapustin [19]. The error caused by truncation of the phase function is significantly reduced by the postprocessing correction in the single scattering [17,16] or the source function integration [5]. The second approach singles out the anisotropic part of the light field without changing the phase function [18,2].

In this paper we compare three different methods. The first one singles out the anisotropic part of radiance, $I_A(\tau,\mu)$, using the Small-Angle Modification [1,6,8]:

$$I(\tau,\mu) = I_A(\tau,\mu) + I_R(\tau,\mu) \quad (5)$$

Importantly, $I_A(\tau,\mu)$ has an analytical expression. With major anisotropy of signal thus removed, the RTE for the smooth regular part, $I_R(\tau,\mu)$, becomes more amenable for the numerical solution than the original Eq. (1). The resulting code DOMAS was described in [12].

In the second method the single scattering approximation is treated as the anisotropic part $I_A(\tau,\mu) = I_1(\tau,\mu)$ [7,20]

$$I(\tau,\mu) = I_1(\tau,\mu) + I_{2+}(\tau,\mu) \quad (6)$$

The second and the higher scattering orders, taken together, represent the regular part in this case: $I_R(\tau,\mu) = I_{2+}(\tau,\mu)$. The computational details of this method, called