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# Scattering of on-axis Gaussian beam by a conducting spheroid with confocal chiral coating

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ABSTRACT

Within the generalized Lorenz–Mie theory framework, an analytical solution to the scattering by a chiral-coated conducting spheroid, for incidence of an on-axis Gaussian beam, is constructed by expanding the incident Gaussian beam, scattered fields as well as internal fields in terms of appropriate spheroidal vector wave functions. The unknown expansion coefficients are determined by a system of linear equations derived from the boundary conditions. Numerical results of the normalized differential scattering cross section are presented, and the scattering characteristics are discussed concisely.



## 1. Introduction

In a chiral or optically active medium, characterized by different phase velocities for the right and left circularly polarized waves, a linearly polarized wave undergoes a rotation of its polarization as it propagates [1–3]. A strong effort has been devoted by so many researchers to the study of electromagnetic wave scattering from chiral bodies. Based on the *T*-matrix method, a theoretical procedure is devised by Lakhtakia et al. to examine the plane wave scattering and absorption characteristics of chiral objects [4]. For normal incidence of a TE or TM plane wave, analytical solutions have been presented by Kluskens and Newman to the scattering by a multilayer chiral circular cylinder [5], and by Khatir et al. to the scattering by a chiral elliptic cylinder [6]. A software package was developed by Demir et al. to calculate the plane-wave scattering by a chiral sphere [7]. For an incident shaped beam, Yokota et al. analyzed the scattering of a Hermite–Gaussian beam by a chiral sphere by using the relations between the multipole fields and the conventional Hermite–Gaussian beam [8]. The generalized Lorenz–Mie theory (GLMT) developed by Gouesbet et al. is effective to describe the electromagnetic scattering of a shaped beam by a dielectric sphere [9–11], and has been extended by Han et al. to a spheroid at parallel incidence [12,13], and by Xu et al. to an arbitrarily oriented, located, and shaped beam scattered by a homogeneous spheroid [14,15]. In this paper, we consider the scattering of an on-axis Gaussian beam by an arbitrarily oriented conducting spheroid with a confocal chiral coating.

The paper is organized as follows. Section 2 provides the theoretical procedure for the determination of the scattered fields of an on-axis Gaussian beam by a conducting spheroid with a confocal chiral coating. In Section 3, numerical results of on-axis Gaussian beam scattering properties are presented. Section 4 is the conclusion.

## 2. Formulation

### 2.1. Expansions of on-axis Gaussian beam and scattered fields in spheroidal coordinates

As illustrated in Fig. 1, a Gaussian beam propagates in free space and from the negative $z'$ to the positive $z'$ axis of the Cartesian coordinate system $O'x'y'z'$, with the

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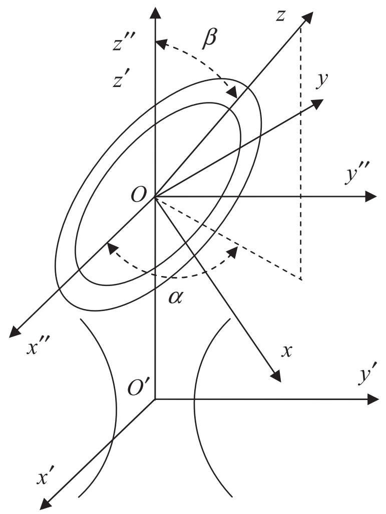


**Fig. 1.** The Cartesian coordinate system $Ox''y''z''$ is parallel to the Gaussian beam coordinate system $O'x'y'z'$, and origin $O$ is on the $O'z'$ axis. The system $Oxyz$ is obtained by a rigid-body rotation of $Ox''y''z''$ through Euler angles $\alpha$ and $\beta$, and a chiral-coated conducting spheroid is natural to $Oxyz$.

middle of its beam waist located at origin $O'$. An accessory system $Ox''y''z''$ is introduced which is obtained by a translation of $O'x'y'z'$ along the $z'$ axis, so that origin $O$ is on the $O'z'$ axis, i.e., at $(0,0,z_0)$ in $O'x'y'z'$ (on-axis case). The system $Oxyz$ is obtained by rotating $Ox''y''z''$ through Euler angles $\alpha$ and $\beta$ [16]. The center of a chiral-coated conducting spheroid is located at origin $O$, and the major axis is along the $z$ axis. The semimajor and semiminor axes are denoted by $a_1$ and $b_1$ for the conducting spheroid and by $a_2$ and $b_2$ for the chiral coating, and the common semi-focal distance by $f$. In this paper, we assume that the time-dependent part of the electromagnetic fields is $\exp(-i\omega t)$.

We have obtained an expansion in [17] of the electromagnetic fields of an on-axis Gaussian beam, for the TM mode, in terms of the spheroidal vector wave functions (SVWFs) with respect to the system $Oxyz$, in the following form:

$$\mathbf{E}^i = E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n [G^m_{n,TE}\mathbf{M}^{r(1)}_{emn}(c) + G'_{n,TE} m\mathbf{M}^{r(1)}_{omn}(c) - iG'_{n,TM} m\mathbf{N}^{r(1)}_{emn}(c) + iG^m_{n,TM}\mathbf{N}^{r(1)}_{omn}(c)] \quad (1)$$

$$\mathbf{H}^i = \frac{E_0}{\eta_0} \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n [-iG^m_{n,TE}\mathbf{N}^{r(1)}_{emn}(c) - iG'_{n,TE} m\mathbf{N}^{r(1)}_{omn}(c) - G'_{n,TM} m\mathbf{M}^{r(1)}_{emn}(c) + G^m_{n,TM}\mathbf{M}^{r(1)}_{omn}(c)] \quad (2)$$

where $c=k_0 f$, $\eta_0=\sqrt{\mu_0/\varepsilon_0}$ (the free-space wave impedance), and the beam shape coefficients $G^m_{n,TE}$, $G'^m_{n,TE}$, $G^m_{n,TM}$ and $G'^m_{n,TM}$ can be expressed explicitly as

$$\begin{bmatrix} G^m_{n,TE} \\ G'^m_{n,TE} \\ G'^m_{n,TM} \\ G^m_{n,TM} \end{bmatrix} = \frac{2(-1)^{m-1}}{N_{mn}} {\sum_{r=2,1}^{\infty}}' \frac{d_r^{mn}(c)}{(r+m)(r+m+1)} \times g_{r+m} \begin{bmatrix} \frac{2}{1+\delta_{m0}} \frac{dP^m_{r+m}(\cos\beta)}{d\beta}\sin\alpha \\ 2m\frac{P^m_{r+m}(\cos\beta)}{\sin\beta}\cos\alpha \\ \frac{2}{1+\delta_{m0}} \frac{dP^m_{r+m}(\cos\beta)}{d\beta}\cos\alpha \\ 2m\frac{P^m_{r+m}(\cos\beta)}{\sin\beta}\sin\alpha \end{bmatrix} \quad (3)$$

where $\delta_{m0}=0$ when $m\neq 0$, and $\delta_{00}=1$. The prime over the summation sign indicates that the summation is over even values of $r$ when $n-m$ is even and over odd values of $r$ when $n-m$ is odd, and $N_{mn}$ are the normalization constants, and $d_r^{mn}(c)$ the expansion coefficients of the spheroidal angle functions $S_{mn}(\eta)$ [17,18]. The $g_{r+m}$ coefficients are for the Gaussian beam expansion in the spherical coordinates with respect to $Ox''y''z''$, and when the symmetrized Davis–Barton model of the Gaussian beam is used [19], can be computed by using the localized approximation as [11]

$$g_{r+m} = \frac{1}{1+2isz_0/w_0}\exp(ikz_0)\exp\left[\frac{-s^2(r+m+1/2)^2}{1+2isz_0/w_0}\right] \quad (4)$$

where $s=1/k_0 w_0$, and $w_0$ is the beam waist radius.

For the TE mode, the corresponding expansions can be obtained only by replacing $G^m_{n,TE}$ in Eqs. (1) and (2) by $-G'^m_{n,TM}$, $G'^m_{n,TE}$ by $G^m_{n,TM}$, $G'^m_{n,TM}$ by $G^m_{n,TE}$, and $G^m_{n,TM}$ by $-G'^m_{n,TE}$.

The scattered fields can be expanded as follows:

$$\mathbf{E}^s = E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n \left[\beta_{mn}\mathbf{M}^{r(3)}_{emn}(c) + \beta'_{mn}\mathbf{M}^{r(3)}_{omn}(c) - i\alpha'_{mn}\mathbf{N}^{r(3)}_{emn}(c) + i\alpha_{mn}\mathbf{N}^{r(3)}_{omn}(c)\right] \quad (5)$$

$$\mathbf{H}^s = \frac{E_0}{\eta_0} \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n \left[-i\beta_{mn}\mathbf{N}^{r(3)}_{emn}(c) - i\beta'_{mn}\mathbf{N}^{r(3)}_{omn}(c) - \alpha'_{mn}\mathbf{M}^{r(3)}_{emn}(c) + \alpha_{mn}\mathbf{M}^{r(3)}_{omn}(c)\right] \quad (6)$$

where $\alpha_{mn}$, $\alpha'_{mn}$, $\beta_{mn}$ and $\beta'_{mn}$ are the unknown expansion coefficients to be determined.

### 2.2. Description of electromagnetic fields within the chiral coating

The constitutive relations for a chiral medium can be written as [1–3]

$$\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E} + i\kappa\sqrt{\mu_0\varepsilon_0}\mathbf{H} \quad (7)$$

$$\mathbf{B} = \mu_0\mu_r\mathbf{H} - i\kappa\sqrt{\mu_0\varepsilon_0}\mathbf{E} \quad (8)$$

where $\kappa$ is the chirality parameter.

The electromagnetic fields in a chiral medium $(\mathbf{E},\mathbf{H})$ are the sum of the right-handed waves $(\mathbf{E}_+,\mathbf{H}_+)$ and left-handed waves $(\mathbf{E}_-,\mathbf{H}_-)$

$$\begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} = \begin{bmatrix} \mathbf{E}_+ \\ \mathbf{H}_+ \end{bmatrix} + \begin{bmatrix} \mathbf{E}_- \\ \mathbf{H}_- \end{bmatrix} \quad (9)$$

where

$$\mathbf{E}_\pm = \pm i\eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}}\mathbf{H}_\pm = \pm i\eta\mathbf{H}_\pm \quad (10)$$