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On the relationship between direct and diffuse radiation

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ABSTRACT

The relationship between the direct and diffuse radiation is analyzed. It is shown that the solution of diffuse radiation is a special case of the solution for direct radiation. No extra effort is needed to find out the solution of diffuse radiation. Under the single-layer condition, the diffuse reflection/transmission/absorption are compared for the two-stream and four-stream approximations. It is found that the δ-four-stream approximation can dramatically improve the accuracy of the δ-two-stream approximation, with relative error generally less than 5%.

1. Introduction

Radiative transfer process is a key issue for remote sensing and climate modeling. In the past several decades, a lot of attention has been paid to the analytical solution of radiative transfer [1–11].

There are two types of solar radiative transfer, and the corresponding results in reflection/transmission are treated separately as the direct and diffuse. The direct radiation is associated with the direct incoming solar beam and the diffuse radiation is associated with the assumed isotropic incoming radiation. Their difference is mainly from the boundary condition.

The diffuse radiation is important because it is used in the multi-layer radiative transfer process based on the doubling/adding method [7,12], which comes from the invariance principle proposed by Chadrasekehar. In addition, the properties of diffuse radiation have been used in the past to provide an estimation for global radiative effect of aerosol [13].

The solutions of radiative transfer for the direct and diffuse radiation are obtained in different ways. A natural question is what is the relationship between the direct and diffuse radiation. Is there any connection between them? In this work we will show, at least for the discrete ordinate method (DOM), the solution of diffuse radiation is a special case for the solution of direct radiation. This helps us to understand the physical essentials of the diffuse radiation.

For the direct radiation, the accuracy of the single-layer reflection and transmission has been demonstrated in [3,7,14] for both of the two-stream and four-stream approximations. For the diffuse radiation, the accuracy of reflection/transmission has been discussed in [15] but only for the two-stream case. In the following, we will show the improvement in accuracy for the diffuse radiation by extending the two-stream approximation to the four-stream approximation.

2. Relationship between the direct and diffuse radiation

The azimuthally averaged radiative transfer equation for the solar intensity $I(\tau,\mu)$ in a plane-parallel atmosphere is

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{\omega}{2}\int_{-1}^{1} I(\tau,\mu')P(\mu,\mu')\,d\mu', \tag{1}$$

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where μ is the cosine of the local zenith angle, τ is the optical depth, ω is the single scattering albedo, $P(\mu,\mu')$ is the azimuthally averaged scattering phase function defining the light incidence at μ' and scattered away at μ, μ_0 is the cosine of the solar zenith angle. The upward and downward fluxes are

$$F^{\pm}(\tau)=2\pi\int_0^1 I(\tau,\pm\mu)\mu\,d\mu.$$

2.1. Direct radiation

For direct radiation, there is a direct solar beam hitting the upper level of the considered layer and there is no upward diffuse radiance at the bottom of the layer. The radiative transfer equation is

$$\mu\frac{dI(\tau,\mu)}{d\tau}=I(\tau,\mu)-\frac{\omega}{2}\int_{-1}^{1}I(\tau,\mu')P(\mu,\mu')\,d\mu', \tag{2a}$$

$$I(0,-\mu)=\delta(\mu,\mu_0)\frac{F_0}{2\pi}, \tag{2b}$$

$$I(\tau_0,\mu)=0, \tag{2c}$$

where τ_0 is the total optical depth of the medium, and F_0 is the solar flux at the top of the layer. The radiation field can be decomposed into two parts:

$$I(\tau,\mu)=I_{dir}(\tau,\mu)+I_{dif}(\tau,\mu),$$

where $I_{dir}(\tau,\mu)$ is the direct solar beam (photons without experiencing scattering), $I_{dif}(\tau,\mu)$ is the diffuse beam (photons experiencing at least one event of scattering). Thus, we have

$$\mu\frac{dI_{dir}(\tau,\mu)}{d\tau}=I_{dir}(\tau,\mu), \tag{3a}$$

$$I_{dir}(0,-\mu)=\delta(\mu,\mu_0)\frac{F_0}{2\pi}, \tag{3b}$$

$$I_{dir}(\tau_0,\mu)=0. \tag{3c}$$

From (2) and (3) the diffuse beam satisfies

$$\mu\frac{dI_{dif}(\tau,\mu)}{d\tau}=I_{dif}(\tau,\mu)-\frac{\omega}{2}\int_{-1}^{1}[I_{dir}(\tau,\mu')+I_{dif}(\tau,\mu')]P(\mu,\mu')\,d\mu', \tag{4a}$$

$$I_{dif}(0,-\mu)=0, \tag{4b}$$

$$I_{dif}(\tau_0,\mu)=0. \tag{4c}$$

The solution of (3) is

$$I_{dir}(\tau,\mu)=0, \tag{5a}$$

$$I_{dir}(\tau,-\mu)=\delta(\mu,\mu_0)\frac{F_0}{2\pi}e^{-\tau/\mu_0}. \tag{5b}$$

Substituting (5) into (4), we have

$$\mu\frac{dI_{dif}(\tau,\mu)}{d\tau}=I_{dif}(\tau,\mu)-\frac{\omega}{2}\int_{-1}^{1}I_{dif}(\tau,\mu')P(\mu,\mu')\,d\mu' -\frac{\omega F_0}{4\pi}e^{-\tau/\mu_0}P(\mu,-\mu_0). \tag{6a}$$

$$I_{dif}(0,-\mu)=0, \tag{6b}$$

$$I_{dif}(\tau_0,\mu)=0. \tag{6c}$$

Eq. (6) is the well-known radiative transfer equation for the diffuse beam of the direct radiation [16]. The exact solution of it is difficult to obtain. Therefore, two-stream and four-stream approximations are widely used to solve (6).

In the two-stream case, the analytical solution of (6) has been obtained and widely used [2,4,6,17]. Here we show the solutions based on [17], since in that the results of direct radiation are the function of diffuse radiation. The direct reflection and transmission are obtained as

$$r(\mu_0)=\frac{F^{+}_{dif}(0)}{\mu_0F_0}=\frac{\mu_1}{\mu_0}[\epsilon(1-\bar{t}e^{-\tau_0/\mu_0})-\gamma\bar{r}], \tag{7}$$

$$t(\mu_0)=\frac{F^{-}_{dif}(\tau_0)}{\mu_0F_0}+e^{-\tau_0/\mu_0} =\frac{\mu_1}{\mu_0}[\gamma(e^{-\tau_0/\mu_0}-\bar{t})-\epsilon\bar{t}e^{-\tau_0/\mu_0}]+e^{-\tau_0/\mu_0}, \tag{8}$$

where

$$\varepsilon=0.5(a+b),\quad \gamma=0.5(a-b),\quad a=-\left(\frac{\omega\mu_0^2}{1-\mu_0^2\kappa^2}\frac{1-\omega g}{\mu_1^2}+3g\right),$$

$$b=\left[\frac{\omega\mu_0^2}{1-\mu_0^2\kappa^2}\frac{(1-\omega)3g\mu_0}{\mu_1}+\frac{1}{\mu_0\mu_1}\right],\quad \kappa=\sqrt{(1-\omega)(1-\omega g)}/\mu_1,$$

and $\mu_1=1/\sqrt{3}$.

$\bar{r}$ and $\bar{t}$ are the reflection/transmission for the diffuse radiation,

$$\bar{r}=\alpha\beta(e^{\kappa\tau_0}-e^{-\kappa\tau_0})/N, \tag{9}$$

$$\bar{t}=(\alpha^2-\beta^2)/N, \tag{10}$$

where $N=\alpha^2e^{\kappa\tau_0}-\beta^2e^{-\kappa\tau_0}$, $\alpha=\sqrt{1-\omega g}+\sqrt{1-\omega}$ and $\beta=\sqrt{1-\omega g}-\sqrt{1-\omega}$.

The four-stream approximation has been discussed in detail by [3,18,16]. Using the 2-node Gaussian quadrature to expand the phase function, the corresponding radiative transfer equation is

$$\mu_i\frac{dI^i_{dif}}{d\tau}=I^i_{dif}-\frac{\omega}{2}\sum_{l=0}^{3}\omega_lP_l(\mu_i)\sum_{j=-2}^{2}I^j_{dif}P_l(\mu_j)a_j -\frac{\omega}{4\pi}F_0e^{-\tau/\mu_0}\sum_{l=0}^{3}\omega_lP_l(\mu_i)P_l(-\mu_0), \tag{11}$$

where $i=\pm1,\pm2$, $I^i_{dif}=I_{dif}(\tau,\mu_i)$, the quadrature point $\mu_{-i}=-\mu_i$ and the weight $a_{-i}=a_i$, with $\mu_1=0.2113248$, $\mu_2=0.7886752$, and $a_1=a_2=0.5$ [19]. The fluxes in the upward and downward directions are defined as

$$F^{\pm}_{dif}(\tau,\mu_0)=2\pi[a_1\mu_1I^1_{dif}(\tau,\pm\mu_1)+a_2\mu_2I^2_{dif}(\tau,\pm\mu_2)] \tag{12}$$

and direct reflection/transmission are

$$r(\mu_0)=\frac{F^{+}_{dif}(0,\mu_0)}{\mu_0F_0}, \tag{13a}$$

$$t(\mu_0)=\frac{F^{-}_{dif}(\tau_0,\mu_0)}{\mu_0F_0}+e^{-\tau/\mu_0}. \tag{13b}$$

The absorption for the direct and diffuse radiation are $a(\mu_0)=1-r(\mu_0)-t(\mu_0)$ and $\bar{a}=1-\bar{r}-\bar{t}$, respectively.