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Optical tomography with the discontinuous Galerkin formulation of the radiative transfer equation in frequency domain

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ABSTRACT

Optical tomography is an inverse method of probing semi-transparent media with the help of light sources. The reconstruction of the optical properties usually employs finite volumes or continuous finite elements formulations of light transport as a forward model for the predictions. In a previous study, we have introduced a generalization of the inversion approach with finite elements formulations by using an integral form of the objective function. The novelty is that the surfaces of the detectors are taken into account in the reconstruction and compatibility is obtained for all finite element formulations. This present paper illustrates this new approach by developing a Discontinuous Galerkin formulation as a forward model for an optical tomography application in the frequency domain framework. Numerical tests are performed to gauge the accuracy of the method in recovering optical properties distribution with a gradient-based algorithm where the adjoint method is used to fastly compute the objective function gradient. It is seen that the reconstruction is accurate and can be affected by noise on the measurements as expected. Filtering of the gradient at each iteration of the reconstruction is used to cope with the ill-posed nature of the inverse problem and to improves the quality and accuracy of the reconstruction.

1. Introduction

Probing biological tissue with the help of near infrared light sources has received a growing attention in the last decade. Compared to classical X-rays tomography, near infrared tomography is harmless as the range of the wavelength of the light source used for probing the tissues is between 600 and 900 nm. This method is a model-based reconstruction technique where the optical properties are recovered from boundary measurements of transmitted light, providing a non-invasive diagnostic tool for medical applications as the optical properties are related to the pathological or physiological state of tissues. The reconstruction is done by a minimization of an objective function based on the errors between the experimental measurements at the detectors and their prediction with a numerical model [1,2]. Hence, an accurate forward model of light transport within the probed medium is needed to achieve good reconstruction of the optical properties distributions.

It is widely accepted that light transport in biological tissue is well described by the radiative transfer equation. However, this equation is a Boltzmann type integro-differential equation that is difficult to solve and analytical solutions are available only for simple cases or with some strong approximations. Different approximations are used, among which the diffusion approximation is

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well suited for high scattering biological tissue [3,4]. While this approximation leads to an equation that is easy to solve, it fails to describe light transport near boundaries, sources, and in void-like region where the mean free path is very large [5]. Therefore, the full radiative transfer equation is to be considered, where the finite volume and finite difference schemes are the most used. These numerical methods are well suited to advection type equations due to their conservative properties, but are less flexible than the finite elements methods in handling complex geometries.

Recently, an increasing interest has been devoted to the finite elements formulations when solving the radiative transfer equation. This tendency in favor of the finite elements method follows from its simplicity, flexibility and property of being able to handle complex geometries and advection type equations. From a standard Galerkin formulation [6], a number of studies have been done to improve the accuracy of these models such as the Streamline Upwind Petrov–Galerkin [7,8], the Least Square formulation [9,10] and the Discontinuous Galerkin formulation [11,12], to name but a few.

Traditionally, optical tomography that employs a model for light transport, involves continuous formulations of the finite elements [13–16]. However, continuous finite element formulations suffer from the lack of local conservativity compared to the discontinuous formulation which allows the use of numerical fluxes to achieve local conservativity [17,18]. In addition, high order accuracy can be achieved with the Discontinuous Galerkin formulation by using higher order polynomial approximation than the finite volume method. The Discontinuous finite elements formulation can be viewed as an all-in-one formulation as with the same formulation, the finite volume method is obtained when constant polynomial elements are used, while using a continuous function yields the standard finite element method [19].

In a comparative study, we have shown that the Discontinuous Galerkin is more accurate than the least square finite element formulation in solving the radiative transfer equation [20]. While the Discontinuous Galerkin formulation is well known to handle strong discontinuities, this method is not suitable for boundary readings because the solutions are not well defined on boundaries due to the discontinuity of the solution field [17]. Then, an extension of the inversion approach to all finite elements formulations was also proposed by using an integral form of the objective function that make the inverse approach with continuous and discontinuous finite elements solution fields [21]. The integral form of the objective function makes possible to take into account the surface of the detectors in optical tomography applications. In this extension [21], it is seen that the integral form has a numerical filtering effect of noise compared to the classical discrete form of the objective function used in optical tomography.

This present paper illustrates this novel approach by developing an optical tomography algorithm where the Discontinuous Galerkin formulation is used as the forward model. Hence, the forward model and its reverse counterpart under the form of its adjoint (co-state) model are employed to recover the distribution of optical properties through a quasi-Newton based minimization of the new definition of the objective function that makes possible the use of Discontinuous finite elements formulation. Numerical tests are performed to gauge the accuracy of the method in recovering optical properties distribution where a gradient filtering is introduced to enhance the contrast of the reconstruction.

The originality of this paper can be summarized in the following points. (i) To the knowledge of the authors, this is the first application of the Discontinuous Galerkin in optical tomography with the full radiative transfer equation through the use of an integral objective function that ensures the compatibility with boundary measurements with discontinuous solutions fields. Through the integral objective function, the surfaces of the detector are taken into account as in practical applications. (ii) Compared to previous studies done in frequency domain optical tomography with the radiative transfer equation, *e.g.* [8,10,12,22], in this study, the collimated source direction is taken into account in the computation of both the forward and the adjoint models. (iii) Filtering of the gradient is used to enhance the reconstruction as a regularization technique acting on the gradient instead of on the function itself as those of Tikhonov types that are classically used in optical tomography applications.

The paper is organized as follows. Section 2 deals with the forward model based on a Discontinuous Galerkin formulation of the collimated irradiation in frequency domain. Section 3 presents the optimization problem, the computation of the minimization of the objective function with respect to the parameters, the equations of the adjoint problem for the computations of the gradient and the reconstruction algorithm. Section 4 presents the numerical tests with according accuracies when synthetic noise is applied with one and two unknown inclusions. Next, Section 5 gives conclusions and perspectives for future developments.

2. Forward model

In optical tomography, the forward model aims at computing the prediction of the boundary reading once the optical properties are known. We present the forward model equation based of the frequency domain radiative transfer equation and the used solution method.

2.1. Radiative transfer in frequency domain

The physical basis of the forward model is the Fourier transform of the radiative transfer equation which writes in a given direction $\vec{\Omega}$ and for each spatial position $r \in \mathcal{D}$ by [1]

$$\left[\vec{\Omega}\cdot\nabla+\frac{i\omega}{c}+\kappa+\sigma_s\right]I(r,\vec{\Omega},\omega)=\frac{\sigma_s}{4\pi}\int_{4\pi}I(r,\vec{\Omega}',\omega)\Phi(\vec{\Omega}',\vec{\Omega})\,d\Omega' \tag{1}$$

where $i=\sqrt{-1}$, c is the light speed in the medium, ω is the modulation frequency, $I(r,\vec{\Omega},\omega)$ is the radiant power per unit solid angle per unit area at the spatial position r in direction $\vec{\Omega}$, $\kappa=\kappa(r)$ and $\sigma_s=\sigma_s(r)$ are respectively the absorption