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Scattering and transversal divergence of an ellipsoidal particle by using Vectorial Complex Ray Model

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A R T I C L E I N F O

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A B S T R A C T

We have developed a novel model – Vectorial Complex Ray Model (VCRM) – for the scattering of a smooth surface objet of arbitrary shape. In this model, a wave is described by bundles of rays, and a ray is characterized not only by its direction and amplitude but also the curvature and the phase of the wave. These new properties allow to take into account the phase shift due to the focal lines of an arbitrary shaped wave and the amplitude due to the divergence/convergence of the wave. The interferences can therefore be calculated correctly for an arbitrarily shaped particle of smooth surface. In this paper, we present an application of the VCRM in the 2D scattering of a plane wave by a homogeneous ellipsoid at oblique incidence. The transversal divergence effect of the wave will be discussed. The rainbows of ellipsoidal droplet and bubble are investigated.

1. Introduction

In the study of electromagnetic and light scattering, the variable separation methods based on the solution of Maxwell equations (or its equivalents) are limited to objects that can be described in a coordinate system of the same geometry, such as sphere, spheroid, ellipsoid, and circular or elliptical cylinder. Even in these "simple" cases, the numerical calculation remains another obstacle. Except for the sphere and the circular cylinder, the size of the scatterer can hardly exceed a few tens of wavelengths. Numerical methods such as T matrix, discrete-dipole approximation (DDA), etc., can be applied to nonspherical particles, but the size parameter of the scatter is also severely limited [1,2].

Geometrical optics is a simple and intuitive method for treating the interaction of an object with electromagnetic or light waves when the dimension of the object is much larger than the wavelength [3–8]. One of its main advantages over other methods is that it can be applied to objects of complex shape, which are hard or even impossible to be dealt with by rigorous theories or most numerical techniques. Many researchers have contributed to the improvement of geometrical optics. Some take into account the forward diffraction or other particular wave effects (Airy theory for the rainbow [3,9,10] and Marston's model for the critical scattering [11,12]). Others combine directly geometrical optics with the electromagnetic wave method [4,13]. However, in these studies interference effects of all order rays are rarely taken into account. In the case of faceted particles, Bi et al. take into consideration the phase information of rays by a solution physical-geometric optics hybrid method [14].

On the other hand, when ray optics is extended to a three dimensional (3D) object of irregular shape, it becomes a heavy task (see [15–18] and references therein) because of the difficulties in the determination of reflection and refraction angles, the calculation of local divergence factors and the phase shift due to focal lines. To overcome these obstacles, we have developed a so-called Vectorial Complex Rays Model (VCRM) [19] that

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includes the wave properties in the ray model. It consists of three points: the rays are dealt with by vectors; the divergence and the focal line phase shifts are calculated by differential geometry; and the total scattered field is the superposition of the contributions of all complex rays. This model makes it possible to calculate the divergence factor of a single ray bundle and is easy to extend to irregularly shaped 3D objects. This model is briefly described in the second section. The validation is made versus Lorenz–Mie theory for spherical particles in the following section. Finally, in the last section, we present an application of the VCRM in the 2D scattering of a plane wave by ellipsoidal droplet and bubble at oblique incidence. The effect of the transversal divergence of the wave will be discussed. The rainbows of an ellipsoidal particle are investigated.

2. Description of the model

In VCRM, the wave is considered as bundles of vectorial complex rays. Each ray denoted by q is characterized by four parameters: amplitude $A_{q\mu}$, phase Φ_q, direction of propagation $\boldsymbol{k}$ and polarization state μ (see [19] for details)

$$\boldsymbol{S}_{q\mu} = A_{q\mu} e^{i\Phi_q} \boldsymbol{k} \tag{1}$$

where $\mu = 1$ and 2 for perpendicular and parallel polarization respectively.

The amplitude of the ray is determined by the Fresnel coefficient $\epsilon_{q\mu}$ and the divergence coefficient $\mathcal{D}_q$

$$A_{q\mu} = \sqrt{\mathcal{D}_q} |\epsilon_{q\mu}| \tag{2}$$

The divergence coefficient $\mathcal{D}_q$ of an emergent ray after $p+1$ time interactions with the dioptry surface is determined by curvature radii of the wave according to

$$\mathcal{D} = \frac{R'_{11}R'_{21}}{R_{12}R_{22}} \cdot \frac{R'_{12}R'_{22}}{R_{13}R_{23}} \cdots \frac{R'_{1,p+1}R'_{2,p+1}}{(r+R'_{1,p+1})(r+R'_{2,p+1})} \tag{3}$$

where r is the distance between the emergent point to the observation point. R_{1q} and R_{2q} $(q = 0, 1, 2, \ldots, p+1)$ are the two principal radii of wavefront curvature of the incident wave, and R'_{1q} and R'_{2q} that of the refracted wave for qth interaction, and p is called the order of the emergent ray. In fact, $R_{1q}R_{2q}$ and $R'_{1q}R'_{2q}$ are respectively the Gauss curvature radii of the incident and refracted waves and they can be determined by the curvature matrix equation [19].

The phase of an emergent ray is composed of four parts: (1) The phase due to optical path Φ_p which can be computed directly according to the optical trajectory. (2) The phase due to the focal point or focal line Φ_f: each time the curvature radius R_{ij} changes the sign we add a phase $\pi/2$, either in or out of the object. (3) The phase due to the reflection or refraction which is included in the Fresnel coefficient Φ_r. (4) The phase of the incident wave Φ_i. The total phase of a ray is then

$$\Phi = \Phi_p + \Phi_f + \Phi_r + \Phi_i + \pi/2 \tag{4}$$

where $\pi/2$ is added for the coincidence with the diffraction field.

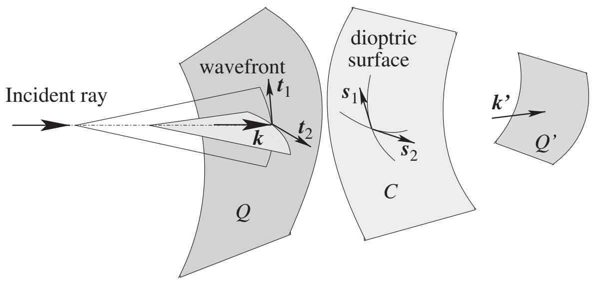

Fig. 1. Schema of the fronts of waves and the dioptric surface.

Consider now an arbitrary wave of wavefront described by its curvature matrix Q impinging on a dioptric surface of curvature matrix C (Fig. 1). The curvature matrix Q' of the wave after refraction or reflection is given by the wavefront matrix equation

$$(\boldsymbol{k}' - \boldsymbol{k}) \cdot \boldsymbol{n} C = k' \Theta'^T Q' \Theta' - k \Theta^T Q \Theta \tag{5}$$

where the letters with prime represent the quantities after refraction or reflection, the superscript T the transpose of the matrix, Θ the projection matrix between the unitary vectors of the coordinates systems on the planes tangent to the wavefront ($\boldsymbol{t}_1$,$\boldsymbol{t}_2$) and the dioptric surface ($\boldsymbol{s}_1$,$\boldsymbol{s}_2$)

$$\Theta = \begin{pmatrix} \boldsymbol{t}_1 \cdot \boldsymbol{s}_1 & \boldsymbol{t}_1 \cdot \boldsymbol{s}_2 \\ \boldsymbol{t}_2 \cdot \boldsymbol{s}_1 & \boldsymbol{t}_2 \cdot \boldsymbol{s}_2 \end{pmatrix}$$

The relation between the wave vector $\boldsymbol{k}$ of the incident ray, the wave vector of reflected or refracted ray $\boldsymbol{k}'$ is determined by the vector Snell law

$$(\boldsymbol{k}' - \boldsymbol{k}) \times \boldsymbol{n} = \boldsymbol{0} \tag{6}$$

where $\boldsymbol{n}$ is the normal of the dioptric surface.

Knowing the amplitude and the phase of each ray, we calculate the total scattered field by the superposition of the complex amplitude of all orders p of the rays emergent from the object and the diffraction

$$E = \sum_{p=0}^{\infty} E_p + E_{diff} \tag{7}$$

The diffraction is calculated in this paper by using the Fraunhofer diffraction of 2D disk which is the projection of the object on a plane perpendicularly to the incident beam.

3. Validation of the model

To validate this model, we have shown theoretically that in the special cases of scattering of a plane wave by a sphere and an infinite circular cylinder at normal incidence, this formalism leads to the classical formulation as given, for example, in [3,9].

A software is realized in CodeGear Delphi 2007 with a friendly interface for ray tracing and calculation of the scattering diagram. To validate our code, the scattered intensities calculated by the software have been compared to the Lorenz–Mie theory (LMT) in the case of spherical particle. As an example, Fig. 2 shows the scattering diagram of a water droplet of radius $a = 20$ μm illuminated