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Near field scattering by multiple infinite cylinders in an absorbing medium at oblique incidence

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ABSTRACT

This paper presents the near field scattering formulation for an arbitrary configuration of closely spaced parallel infinite circular cylinders in an absorbing/magnetic medium at oblique incidence. No restriction is placed on the polarization of the incident plane wave, and complex permeability and refractive index are allowed for the cylinders and host medium. The electric and magnetic fields and Poynting vector of the scattered radiation are derived for both the near and far fields. Near field scattering by an assembly of cylinders at oblique incidence is illustrated for a transverse magnetic, transverse electric, and elliptically polarized source waves.

1. Introduction

Electromagnetic scattering by small particles is of significant interest to research in atmospheric scattering, remote sensing, and radiative transfer. Particles in many physical systems can be approximated as spheres or infinite cylinders, for which the exact solutions of Maxwell's equations have been well established [1,2]. Problems in atmospheric radiation and astrophysics usually involve spheres that are treated by the Mie theory [3]. On the other hand, cylindrical particles in materials and biological tissues are commonly approximated as infinite cylinders due to the large length-to-diameter and -wavelength ratios [4,5]. The solution for infinite cylinder provides the basis for treating radiative transfer in fiber composites and biological tissues [6–9].

Scattering by multiple infinite cylinders is important for the study of electromagnetic and acoustic wave propagations in high density media. Many studies treating multiple cylinders in a non-absorbing medium over the past several decades up to 2006 can be found in [10]. Most studies treated only perpendicular incidence such that the incident wave propagates normal to the axes of the cylinders. Oblique incidence on multiple homogeneous cylinders was first treated by Lee [11] and recently by Henin et al. [12] and Yan et al. [13], and the generalization to multiple layered cylinders was developed by Lee [14]. These studies focused on far-field scattering for cylinders in free space or a non-absorbing host medium.

For composite materials and biological tissues, the refractive index of the host medium can exhibit considerable spectral variation from purely real to complex values at different wavelengths. The scattering solution involving an absorbing host medium was reported in [15,16] for perpendicular incidence and [17] for oblique incidence on a single cylinder, and [18] for oblique incidence on multiple nonhomogeneous cylinders, which were all concerned with the far field behavior. The far-field scattering properties for single and multiple cylinders given in [15–18] can be applied to radiative transfer simulation. Phenomena such as photonic nanojets and local field enhancement occur in the close proximity of the cylinders [19,20], which must be treated by utilizing the electric and magnetic fields in the near field. Near field scattering by multiple infinite

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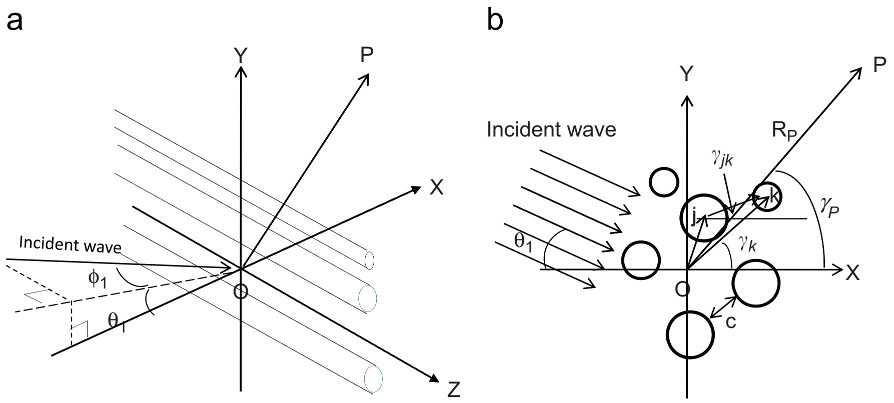

Fig. 1. Schematic diagrams depicting the geometry of the present problem: (a) isometric view and (b) side view.

cylinders in an absorbing medium was recently treated by Lee [21], and in non-absorbing medium by Shafer et al. [22]. However, both studies were restricted to perpendicular incidence on the cylinders.

The present study focuses on near field interactions that include dependent scattering between closely spaced parallel cylinders. In this paper we generalize the near field solution to oblique incidence on multiple infinite cylinders embedded in an absorbing medium. The refractive index and permeability of both the cylinders and host medium can be complex, and the polarization of the source wave is non-restrictive. Numerical results are presented to illustrate near field scattering by an assembly of closely spaced cylinders in an absorbing/magnetic host medium at oblique incidence.

2. Theory

Fig. 1(a,b) shows the schematic diagrams of the present problem. The source wave emanates at O'XYZ and propagates in the direction specified by polar and azimuth angles (ϕ_1,θ_1) at an arbitrary configuration of infinite cylinders. The cylinders are aligned parallel to the Z-axis, and the location of cylinder j is designated by the radial vector $\vec{R}_j$ and azimuth angle γ_j with respect to the reference frame OXYZ. The electric (E) and magnetic (H) fields in the medium satisfy Maxwell's relations:

$$\vec{E} = \nabla\times(\vec{e}_z v) + \frac{i}{k}\nabla\times\nabla\times(\vec{e}_z u) \tag{1}$$

$$\vec{H} = -\frac{\tilde{m}}{\mu}\nabla\times(\vec{e}_z u) + \frac{i}{\mu k_o}\nabla\times\nabla\times(\vec{e}_z v) \tag{2}$$

where u and v are commonly called the Hertz potentials, k_o is the free space propagation constant, and

$$(\tilde{m},\mu,k) = \begin{cases} (\tilde{m}_1,\mu_1,k_1=\tilde{m}_1 k_o), & R_{jP} > r_j \\ (\tilde{m}_j,\mu_j,k_j=\tilde{m}_j k_o), & R_{jP} < r_j \end{cases} \tag{3}$$

are the refractive index, permeability, and propagation constant, respectively, and r_j is the radius of cylinder j. For generality we consider complex permeability and refractive index for the cylinders and host medium.

2.1. Source incident wave

The Hertz potential $\psi\in(u,v)$ of the incident wave at point P relative to cylinder j is given by

$$\psi_j^{inc} = \varepsilon_j\alpha_\psi \exp(-i\vec{k}_1\cdot\vec{R}_{jP} - ih_1 z) \tag{4}$$

where $\varepsilon_j = \exp(-i\vec{k}_1\cdot\vec{R}_j)$ is the phase shift of cylinder j from OXYZ, α_ψ is the complex amplitude at OXYZ, $h_1 = k_1\sin\phi_1$, and

$$\vec{k}_1 = k_1(\cos\phi_1\cos\theta_1\vec{e}_x - \cos\phi_1\sin\theta_1\vec{e}_y + \sin\phi_1\vec{e}_z) \tag{5}$$

is the complex propagation constant of the host medium. Utilizing Eq. (4) in (1)–(2) yields the E and H fields of the incident wave on cylinder j as

$$\vec{E}_j^{inc} = i\varepsilon_j\ell_1(\alpha_v\vec{P}_M^{inc} + \alpha_u\vec{P}_N^{inc})\exp(-i\vec{k}_1\cdot\vec{R}_{jP} - ih_1 z) \tag{6}$$

$$\vec{H}_j^{inc} = i\varepsilon_j\ell_1\frac{\tilde{m}_1}{\mu_1}(-\alpha_u\vec{P}_M^{inc} + \alpha_v\vec{P}_N^{inc})\exp(-i\vec{k}_1\cdot\vec{R}_{jP} - ih_1 z) \tag{7}$$

where $\ell_1 = k_1\cos\phi_1$, and the unit vectors are given by

$$\vec{P}_M^{inc} = \sin\theta_1\vec{e}_x + \cos\theta_1\vec{e}_y \tag{8}$$

$$\vec{P}_N^{inc} = -\sin\phi_1\cos\theta_1\vec{e}_x + \sin\phi_1\sin\theta_1\vec{e}_y + \cos\phi_1\vec{e}_z \tag{9}$$

$$\vec{P}_M^{inc}\times\vec{P}_N^{inc} = \vec{k}_1/k_1 = \vec{e}_1 \tag{10}$$

The polarization of the source wave is dictated by the complex amplitudes (α_u,α_v). It is linearly polarized if the phase difference between α_u and α_v is a multiple of π, circularly polarized if $|\alpha_u| = |\alpha_v|$ and the phase difference is an odd multiple of $\pi/2$, and elliptically polarized if $|\alpha_u| \neq |\alpha_v|$ and the phase difference is not an even multiple of $\pi/2$. Specifying $(\alpha_u=1,\ \alpha_v=0)$ yields the transverse magnetic (TM) mode, and $(\alpha_u=0,\ \alpha_v=1)$ gives the transverse electric (TE) mode.

The Poynting vector of the incident wave at cylinder j is obtained as

$$\vec{S}_j^{inc} = \frac{c_o}{8\pi}\mathrm{Re}\left(\vec{E}_j^{inc}\times\vec{H}_j^{inc*}\right)$$