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Small-angle modification of the radiative transfer equation for a pseudo-spherical atmosphere

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ABSTRACT

The conventional pseudo-spherical technique relies on the separation of the total radiance into the direct solar beam and the diffuse radiance; the direct solar radiance is treated in a spherical geometry, while the diffuse radiance is computed in a plane-parallel geometry. In the small-angle modification of the radiative transfer equation, the total radiance is separated into an anisotropic part and a regular part. In this paper, we pOresent two formulations of the small-angle modification of the radiative transfer equation for a pseudo-spherical atmosphere. In the first formulation, we solve the radiative transfer equation for the diffuse radiance in a pseudo-spherical atmosphere with an additional anisotropic source term computed in a plane-parallel atmosphere, while in the second formulation we solve the radiative transfer equation for the regular solution in a plane-parallel atmosphere with an additional pseudo-spherical correction term. The numerical analysis revealed that the accuracy of the small-angle models is acceptable.

1. Introduction

Nowadays the amount of satellite remote sensing data is increasing due to the state-of-the-art instruments with high spatial and spectral resolution like GOME-2. A massive amount of data is expected from the new generation of European atmospheric composition sensors: Sentinel 5 Precursor, Sentinel 4 and Sentinel 5. The atmospheric Sentinel sensors will generate two orders of magnitude more data than GOME-2. The fast processing and interpretation of such data is a key component for new applications like air quality monitoring and chemical weather prediction. The operational systems required to process these satellite data need to be optimized in order to cope with near-real-time requirements. An important aspect of trace gas retrievals is that such retrievals are hindered by the presence of clouds. For this reason, the ozone retrieval from the GOME-2 instrument, for example, is performed in two steps [1]. In the first step, the cloud parameters (fractional cover, cloud top height, and cloud optical thickness) are mainly obtained in the Oxygen A-band (758–778 nm), while in the second step, the cloud information is used to retrieve ozone column densities in a spectral window from 325 nm to 335 nm of the O_3 Huggins absorption band. In both steps, the retrieval algorithm uses the same radiative transfer model, in which the radiative transfer equation is solved for a cloudy atmosphere in the pseudo-spherical approximation of the direct beam attenuation [2,3]. To increase the efficiency of the GOME-2 data processor, a Lambertian reflecting surface, with a surface albedo taken from the GOME/TOMS LER data set [1], is considered. As the forward model simulations are by far the more time consuming part of the retrieval systems, fast and accurate alternatives to solving the radiative transfer equations in the pseudo-spherical approximation and for a cloudy atmosphere are urgently needed.

In the conventional Milne–Chandrasekhar method [4], the singularity in the angular distribution of the total radiance is removed by splitting the total radiance I into

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the direct solar beam I_{sun} and the diffuse radiance I_{d}. The phase functions of large particles such as cloud droplets are strongly anisotropic due to the strong peak in the forward scattering direction. The treatment of highly anisotropic phase functions, expressed by a Legendre polynomial expansion, creates serious numerical difficulties in computing the diffuse radiance. The reason is that a large number of Legendre terms are required for accurate diffuse radiance calculations. Different approaches were designed to accelerate the solution of the radiative transfer equation without significant loss of accuracy. In the delta-M method [5], the exact phase function is approximated by a linear combination of a delta-function and a truncated phase function. In this way, the original transfer equation with a strong asymmetry parameter is reduced to a more tractable problem with less anisotropic phase function.

An alternative method consists in the elimination of the anisotropic part of the total radiance for further discretization. This idea has evolved from the Eddington–Milne approach [6], in which the anisotropic part is due to the non-scattered radiation in the small-angle modification of the radiative transfer equation [7,8], in which the anisotropic part is described by the Goudsmit–Saunderson solution [9].

In the latter approach, the total radiance I is splitted into an anisotropic part I_{a}, which solves the radiative transfer equation under the small-angle approximation, and a regular (or a smooth) part I_{r}. Thus, we can write

$$I = I_{\mathrm{sun}} + I_{\mathrm{d}} = I_{\mathrm{a}} + I_{\mathrm{r}}. \quad (1)$$

If the small-angle solution I_{a} is extracted from total radiance I accurately enough, then for strongly anisotropic scattering, the regular part I_{r} is much smoother than the diffuse radiance I_{d}. Recently, a number of papers have been devoted to the analysis of the small-angle modification of the radiative transfer equation for a homogeneous plane-parallel layer [10–12].

The goal of this paper is to extend the small-angle modification of the radiative transfer equation for a pseudo-spherical atmosphere, and to investigate the possible advantages of this method over the conventional approach. One of the main underlying aims of this work is the design of fast and accurate radiative transfer models for the retrieval of atmospheric constituents from satellite-measurements of earthshine radiation.

The paper is organized as follows. Section 2 is devoted to the theoretical description of the small-angle modification of the radiative transfer equation for a pseudo-spherical atmosphere, while Section 3 deals with a numerical analysis of the method. Basically, we estimate the plane-parallel model errors and analyze the accuracy of the small-angle approach. In a companion paper [15], we will analyze the efficiency of this approach through comparisons against a conventional discrete ordinate model.

2. Mathematical formulation

In a plane-parallel atmosphere with a lower Lambertian reflecting surface, the boundary-value problem for the total radiance $I(r,\Omega)$ at the generic point r and in the direction Ω, reads as

$$\begin{cases} \mu \dfrac{\mathrm{d}I}{\mathrm{d}r}(r,\Omega) = -\sigma_{\mathrm{ext}}(r) I(r,\Omega) + \dfrac{\sigma_{\mathrm{sct}}(r)}{4\pi} \int_{4\pi} P(r,\Omega,\Omega') I(r,\Omega')\,\mathrm{d}\Omega', \\ I(r_{\mathrm{TOA}}, \mu < 0, \varphi) = F_{\mathrm{sun}} \delta(\mu + \mu_0) \delta(\varphi - \varphi_0), \\ I(r_{\mathrm{s}}, \mu > 0, \varphi) = \dfrac{A}{\pi} \int_0^{2\pi} \int_0^1 I(r_{\mathrm{s}}, -\mu, \varphi) \mu \,\mathrm{d}\mu \,\mathrm{d}\varphi. \end{cases} \quad (2)$$

Here, σ_{ext} and σ_{sct} are the extinction and scattering coefficients, respectively, F_{sun} is the incident solar flux, P is the scattering phase function, and A is the surface albedo. The points r_{TOA} and r_{s} correspond to the top of the atmosphere and the Lambertian reflecting surface, respectively. The direction Ω is characterized by the cosine of the zenith angle $\mu = \cos\theta$ and the azimuthal angle φ, and we indicate this dependency by writing $\Omega = (\mu, \varphi)$. Our convention is that $\mu = 1$ for upward radiation and $\mu = -1$ for downward radiation. In this regard, the incident solar direction is written as $\Omega_0 = (-\mu_0, \varphi_0)$ with $\mu_0 > 0$.

Before proceeding we note that the direct solar beam, solving the boundary-value problem

$$\begin{cases} \mu \dfrac{\mathrm{d}I_{\mathrm{sun}}}{\mathrm{d}r}(r,\Omega) = -\sigma_{\mathrm{ext}}(r) I_{\mathrm{sun}}(r,\Omega), \\ I_{\mathrm{sun}}(r_{\mathrm{TOA}}, \mu, \varphi) = F_{\mathrm{sun}} \delta(\mu + \mu_0) \delta(\varphi - \varphi_0), \end{cases} \quad (3)$$

can be expressed as a spherical harmonics expansion

$$\begin{aligned} I_{\mathrm{sun}}(r,\Omega) &= F_{\mathrm{sun}} \mathcal{T}_{\mathrm{p}}(r) \delta(\mu + \mu_0) \delta(\varphi - \varphi_0) \\ &= \frac{F_{\mathrm{sun}}}{2\pi} \mathcal{T}_{\mathrm{p}}(r) \sum_{n,m} (2 - \delta_{m0}) P_n^m(\mu) P_n^m(-\mu_0) \cos[m(\varphi - \varphi_0)], \end{aligned} \quad (4)$$

where $\mathcal{T}_{\mathrm{p}}(r)$ is the transmission between the top of the atmosphere and the generic point r,

$$\mathcal{T}_{\mathrm{p}}(r) = \exp\left[-\frac{1}{\mu_0} \int_r^{r_{\mathrm{TOA}}} \sigma_{\mathrm{ext}}(r')\,\mathrm{d}r' \right], \quad (5)$$

$(1/\mu_0) \int_r^{r_{\mathrm{TOA}}} \sigma_{\mathrm{ext}}(r')\,\mathrm{d}r'$ is the solar optical depth, P_n^m are the associated Legendre functions, the index "p" refers to the plane-parallel atmosphere, and the simplified notation $\sum_{n,m}$ should be understood as $\sum_{n=0}^{\infty} \sum_{m=0}^{n}$.

To extend the small-angle modification of the radiative transfer equation to a pseudo-spherical atmosphere we adopt the following strategy. Using the analytical representation of the anisotropic solution in a plane-parallel atmosphere, we formulate an appropriate boundary-value problem for the regular solution in a pseudo-spherical atmosphere, and solve the problem numerically in the framework of the discrete ordinate method.

2.1. Anistropic solution in a plane-parallel atmosphere

The derivation of the anisotropic solution in a plane-parallel atmosphere is a standard approach and can be found, for example, in [10–12]. The anisotropic term I_{a} solves the boundary-value problem

$$\begin{cases} -\mu_0 \dfrac{\mathrm{d}I_{\mathrm{a}}}{\mathrm{d}r}(r,\Omega) = -\sigma_{\mathrm{ext}}(r) I_{\mathrm{a}}(r,\Omega) + \dfrac{\sigma_{\mathrm{sct}}(r)}{4\pi} \int_{4\pi} P(r,\Omega,\Omega') I_{\mathrm{a}}(r,\Omega')\,\mathrm{d}\Omega', \\ I_{\mathrm{a}}(r_{\mathrm{TOA}}, \mu, \varphi) = F_{\mathrm{sun}} \delta(\mu + \mu_0) \delta(\varphi - \varphi_0), \end{cases} \quad (6)$$