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Radiation propagation in random media: From positive to negative correlations in high-frequency fluctuations

Anthony B. Davis [a,*], Mark B. Mineev-Weinstein [b,1]

[a] Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109, USA
[b] Los Alamos National Laboratory, Solid Mechanics and Material Properties (X-1-SMMP), Los Alamos, NM 87545, USA

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ABSTRACT

We survey research on radiation propagation or ballistic particle motion through media with randomly variable material density, and we investigate the topic with an emphasis on very high spatial frequencies. Our new results are based on a specific variability model consisting of a zero-mean Gaussian scaling noise riding on a constant value that is large enough with respect to the amplitude of the noise to yield overwhelmingly non-negative density. We first generalize known results about sub-exponential transmission from regular functions, which are almost everywhere continuous, to merely "measurable" ones, which are almost everywhere discontinuous (akin to statistically stationary noises), with positively correlated fluctuations. We then use the generalized measure-theoretic formulation to address negatively correlated stochastic media without leaving the framework of conventional (continuum-limit) transport theory. We thus resolve a controversy about recent claims that only discrete-point process approaches can accommodate negative correlations, i.e., anti-clustering of the material particles. We obtain in this case the predicted super-exponential behavior, but it is rather weak. Physically, and much like the alternative discrete-point process approach, the new model applies most naturally to scales commensurate with the inter-particle distance in the material, i.e., when the notion of particle density breaks down due to Poissonian—or maybe not-so-Poissonian—number-count fluctuations occur in the sample volume. At the same time, the noisy structure must prevail up to scales commensurate with the mean-free-path to be of practical significance. Possible applications are discussed.

1. Introduction, context, and overview

Although radiative transfer theory is highly developed for uniform or slowly varying optical media, natural optical media are often variable at all observable scales. Clouds are a good example where in situ probing by aircraft show highly variable extinction as well as liquid water content. Being highly turbulent dynamical environments, the fluctuations of an admixture such as condensed water particles in clouds ($\sim$10 s of μm in size) are expected over a huge range of scales: down to the Kolmogorov dissipation scale ($\sim$a few mm) and up to the cloud-system scale ($\sim$10 s of km). Somewhere in this range are the radiatively relevant scales such as the mean-free-path, or e-folding distance, for solar (thermal) radiation propagation between (emission,) scattering, absorption, or escape events in the transport process. As contrived as they are, nuclear engineering systems (e.g., reactors) can also exhibit macroscopic cross-section variations over a very wide range of scales. So wide as to challenge the memory requirements and the related efficiency of the most detailed computational transport models.

There is an obvious qualitative difference between these two examples. A priori, cloud structure is inherently random except maybe at the largest scales where, for instance, clouds often exhibit strong gradients in the vertical. By contrast, we

* Corresponding author. Tel.: +1 818 354 0450; fax: +1 818 393 4619.
E-mail address: Anthony.B.Davis@jpl.nasa.gov (A.B. Davis).
[1] Now at: Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, 01187 Dresden, Germany.

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envision nuclear systems as carefully designed down to the smallest detail. And then there are pebble-bed reactors, a relatively new concept, where small (~6 cm diameter) spherical pellets are stacked randomly in a large vessel; these pellets are themselves made of a graphite shell filled with tiny (~1 mm diameter) spheres of fuel surrounded by other materials.[2] Here again, the scales of variability straddle the neutron mean-free-paths of the various materials, including the fluid between the pellets.

In short, transport media whose material properties can only be described in practice by statistical methods are playing increasingly important roles in atmospheric radiative transfer (e.g., the role of clouds in the large-scale radiative energy budget), in nuclear engineering (e.g., above-mentioned next-generation reactors or fractures in shielding materials), in medical physics (e.g., dosimetry and computed tomography), in astrophysics (e.g., convectively unstable stars), and so on. If the 3D spatial structure of the optical medium can only be described in probabilistic terms, then one can only ask of the corresponding transport theory to deliver domain-average quantities. Two broad classes of solutions have emerged for such transport problems: "homogenization" and, broadly speaking, "alternate transport theories." In the former pursuit, one seeks ways of redefining the material properties of the medium, as if it were uniform at the (usually large) scales of interest, but in a manner that accounts for the dominant effects of smaller scale (usually unresolved) variability. In the later approach, one arrives at new transport equations, to be solved analytically or numerically.

Homogenization (a.k.a. the "effective medium" approach) is very attractive because it reduces the difficult multi-dimensional problem to a much simpler problem for a uniform medium, which has known solutions (at least in 1D, using slab geometry). For examples in the atmospheric literature, see Davis et al. [1], Cahalan et al. [2,3], Cairns et al. [4], and Petty [5]. For examples from nuclear engineering, see Graziani [6] and Olsen et al. [7,8].

Although they pose new technical challenges, new transport equations describing the stochastic transport problem are generally a more realistic approach. A well-known example is the theory of transport in Markovian binary mixtures, which is expressed as a pair of coupled integro-differential transport equations; it has been surveyed in great depth by Pomraning [9], Byrne [10], and Kassianov and Lane [11, in this Special Issue], respectively, from the particle transport, radiative transfer (RT), and broader perspectives. There are other such mean-field transport theories, for instance, Stephens [12] reconsiders the classic two-stream model in 1D RT in a manner that incorporates some 3D RT phenomenology uncovered in his numerical simulations [13]. As another example, Davis and Marshak [14–16] have developed diffusion and transport theories, where the particle free-path distributions have power-law tails to represent the mean propagation kernel for heterogeneous media.

Between homogenization theories and alternative transport equations, there is an intermediate approach to transport in stochastic media, very popular in the atmospheric community, is the independent pixel (or column) approximation—the IPA (or ICA). Therein, one solves the 1D RT problem for given optical properties, but one or more of these parameters are actually random variables with given probability density functions (PDFs). Typically, the optical depth of the medium is varied. In the IP(C)A, one simply averages the outcome of the 1D RT computation weighted by the known PDF. Although the concept goes at least as far back as the 1972 report by Mullamaa et al. [17], the terminology was introduced in the mid-1990's [18,19]. The IP(C)A was used originally to derive closed-form expressions, including one or more new parameters for the variability, but more recently it has been implemented numerically, particularly with global climate models in mind [20–22].

There is an interesting and important question about transport in random optical media that is more elementary than all of the above solutions, which is simply to characterize propagation between emission, scattering, absorption, and detection/escape events. Studies are on-going, for instance, in chord-length distributions[3] for media made of closely packed disks or spheres [7,8]. We see this question as one about the prevailing law of direct transmission, which is closely related to the PDF for the free paths covered by the transported particles. Imagining a (pulsed) point source, how many particles are stopped near (early) versus far (late)? The standard answer is: an exponential distribution, the famous Bouguer–Lambert–Beer law in radiometry. However, that answer, completely determined by the mean-free-path (MFP), applies only to strictly uniform media.

A recent series of publications addressing this fundamental issue have provoked some controversy about non-exponential transmission laws, largely because of the unconventional description of the propagation part of transport problem in terms of "discrete-point process" theory rather than the traditional "continuum" theory encapsulated in the linear Boltzmann equation. Introducing discrete-point process modeling into transport through heterogeneous media, Kostinski [23] argued strongly that the presence of spatial correlations in *statistically* homogeneous media will invariably lead to sub-exponential behavior in the law of direct transmission. His findings were critiqued by Borovoi [24] who relied on classic continuum theory. In his reply, Kostinski [25] insists that the discrete-point approach is more fundamental and, to rest his case, he claims that transport in negatively correlated (a.k.a. "super-homogeneous") media can be modeled by discrete-point methods but *not* by continuum methods, pointing to a more detailed study by Shaw et al. [26]. One of the present authors weighed in very strongly favoring non-exponential mean transmission laws for *positively* correlated media using mainstream/continuum-based radiation transport theory [15]. This leaves

[2] See http://en.wikipedia.org/wiki/Pebble_bed_reactor, or http://www.cd-adapco.com/press_room/case_studies/060118_pebblebed.html for more information.

[3] These statistics are of immediate interest for stochastic RT in binary Markovian media.