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Refractive index imaging from radiative transfer equation-based reconstruction algorithm: Fundamentals

Joan Boulanger [a,*], Olivier Balima [b], André Charette [b]

[a] Gas Turbine Laboratory, Building M-10, Montréal Road Campus, Institute for Aerospace Research, National Research Council, Ottawa, Ontario, Canada K1A 0R6
[b] Groupe de Recherche en Ingénierie des Procédés et Systèmes, Université du Québec à Chicoutimi, QC, Canada G7H 2B1

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ABSTRACT

We present the first reconstruction algorithm for refractive index imaging, which is based on the radiative transfer equation (RTE). An objective function is iteratively minimized to find a solution to the problem of inversion of the refractive index field. The function describes the discrepancies of the emerging light measurements on the surface of the sample to be probed with predicted data from the corresponding numerical model. The unknown refractive index field is updated within each reconstruction iteration according to a search direction on the index distribution given by the adjoint model to the RTE. In this paper, emphasis is placed on the theoretical aspects. Preliminary tests are demonstrated on generic phantoms.

1. Introduction

The romantic image of 'shadows dancing in fire' is evidence that spatial variation of refractive index has long been helpful to mankind in visualizing non-isothermal flows. Those shadows and distorted pictures created by refraction are due to the fact that hot air or combustion products have a different refractive index than the surrounding cold air, as described by the Gladstone–Dale equation, which linearly relates index variation to gas density. A hot pocket of air in a cold surrounding has a slightly lower refractive index. Should a ray of light crossing the space in the neighbourhood of the interface between the hot and cold air, it will enter a zone of refractive index gradient perpendicular to its direction. According to the Fermat principle it will bend its path. Therefore, for an observer on the other side, some rays of light coming from the background do not arrive at the proper place and the picture looks distorted and shadowed. This is the principle of flow visualization by the so-called shadowgraphy. This was noticed in the seventeenth century by the famous scientist R. Hooke, *Micrographia*, Observation LVIII, who built laboratory devices to exploit this flow optical imaging opportunity and demonstrated it at the Royal Academy of Sciences. It became known as Schlieren imaging. This phenomenon and the direct visualization of flow it permits were also noted by the physician J.-P. Marat one century later in his *Recherches Physiques sur le Feu* (before he embarked upon a somewhat more infamous second life during the French Revolution). However, Schlieren imaging provides only a qualitative projection picture and research is underway to overcome this limitation, such as development of the BOS technique [1].

Regarding dense media probing, optical tomography is also receiving interest in the refractive index as an optical variable to be reconstructed, e.g. [2] where it has indeed been found that refractive index offers a good sensitivity for reconstruction. In this context, the studies dedicated to model-based inversion of a refractive index distribution employ the diffusion approximation (DA) to the radiative transport equation (RTE). The DA is a convenient tool,

* Corresponding author. Tel.: +1 613 949 7687; fax: +1 613 952 7677.
E-mail address: Joan.Boulanger@nrc-cnrc.gc.ca (J. Boulanger).
URL: http://www.nrc-cnrc.gc.ca/eng/facilities/iar/aerodynamics-combustion.html (J. Boulanger).

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computationally efficient, but suffers from numerous drawbacks as it is unsuitable for poorly scattering media such as areas with low scattering, or areas close to boundaries. Furthermore, in their comparison with the Monte-Carlo method as a benchmark, Brewster and Yamada [3] indicated that the DA fails in predicting the maximum hemispherical reflectance, which surges quickly after an ultra-short laser pulse.

It is thus important to look forward using the full RTE in the reconstruction process of refractive index distributions. The first pioneering treatment of radiation transport of interest for the present work is that published by Lemonnier and Le Dez [4]. They proposed a solution for the RTE in its so-called local form. It means that the radiation field was solved in an Eulerian manner with two streaming operators: the conventional one for the convection of the light intensity along the ray direction, and a new one allowing a transfer of the light energy from one ray direction to its neighbour, simulating the bending of the ray, according to the Fermat principle. The derivation of the RTE in its local form accounting for varying refractive index has subsequently been generalised by Liu in [5]. Their starting point was the RTE in its most general radiosity form in a curvilinear system that is rewritten with the help of the ray optics equation to make the streaming operators appear. We are indebted to all these developments for the building of our RTE-based optical refractive index imaging algorithm presented below. It must be added that an inverse problem based on a 1-D RTE with variable refractive index as found in [4] has been achieved. The reconstruction dealt with a source term field [6]. However, according to the literature, no attempt has been made in inverting a refractive index distribution with a model-based algorithm relying on the RTE and this study explores this approach. Technically, the novelty is related to the link between the refractive index and the hyperbolicity of the RTE, in contrast to other optical parameters such as absorption and scattering, which appear in source terms.

In the following we present the development of the model-based inversion methodology, employing the RTE and its adjoint, to iteratively diminish a least-square error functional of the transmitted/scattered light measurements by a gradient method. The forward model is briefly recalled involving the RTE in media of varying index, while the inverse model is presented in detail. Simple numerical tests are used to illustrate these theoretical developments.

2. Development

The transient RTE (tRTE) is a Boltzmann type integro-differential equation, which accounts for the convection at the speed of light in the material of the directional radiation intensity I, its extinction along its path in the material, and its coherent elastic scattering redistribution of the photons in the other directions of the extinguished fraction that has not been absorbed. This directional intensity also receives contributions from the other scattered directions, in situ sources such as material emission, boundary inputs, and inelastic scattering from the intensity fields with different wavelengths. In the following, we shall focus on a monochromatic formalism with coherent scattering and no source terms to shorten the presentation. Note also that the intensity direction is bent when it travels across a zone with varying refractive index, which is at the core of this study. Therefore, the corresponding equation is

$$\frac{n^2}{c}\frac{D}{Dt}\left[\frac{I(s,t)}{n^2}\right] = -(\kappa+\sigma_s)I(s,t)+\frac{\sigma_s}{4\pi}\int_{4\pi} d\Omega' I(s',t)\boldsymbol{\Phi}(\vec{\Omega}',\vec{\Omega}) \tag{1}$$

where t is the time, s the abscissa on the ray trajectory $\vec{\Omega}$, and n, c, κ, σ_s, and Φ are the refractive index, speed of light in vacuum, absorption coefficient, scattering coefficient, and scattering phase function, respectively. Details on these physical quantities may be found in [7]. The boundary condition is given by

$$I_{\cos(\vec{\Omega},\vec{n})<0} = \int_{\cos(\vec{\Omega}',\vec{n})>0} d\Omega' \rho(\vec{\Omega}',\vec{\Omega}) I \cos(\vec{\Omega}',\vec{n}) \tag{2}$$

where $\rho(\vec{\Omega}',\vec{\Omega})\cos(\vec{\Omega}',\vec{n})$ is the ratio of reflection of the intensities in the outgoing direction $\vec{\Omega}'$ and the incoming direction $\vec{\Omega}$ at the frontier of the phantom, and $\vec{n}$ is the outgoing normal. For clarity of the writing, I has lost its arguments. To shorten the following developments, ρ will be taken as strictly null, without loss of generality.

The total derivation in the left-hand side of Eq. (1) is split as follows:

$$\frac{n^2}{c}\frac{D}{Dt}\left[\frac{I}{n^2}\right] = \frac{n}{c}\frac{\partial I}{\partial t}+n^2\frac{\partial}{\partial s}\left[\frac{I}{n^2}\right] \tag{3}$$

for $\partial s/\partial t = c/n$ along the $\vec{\Omega}$ direction. The derivative with respect to s is expanded in turn to highlight the differentiation with respect to the whole space coordinates: x, y, and z for a Cartesian mesh (which, following the usual methodology, leads to the associated direction cosines μ, η, and ξ) and two additional variables, θ and ϕ, the familiar coordinates in the spherical frame describing the angular dependence of the light intensity I. The differentiation with respect to both these variables is necessary for the case where a ray is bent and changes its angular direction, as happens in media of varying refractive index. The local ray equation of geometric optics, $(d/ds)(n(d\vec{r}/ds)) = \vec{\nabla} n$, with $\vec{r}$ the position vector, provides the relationship between the rate change of the direction cosines along the path s and the local gradient of refractive index. Then, with the expression of the direction cosines with respect to θ and φ, the derivative of the latter with respect to s can be made explicit. After some algebra whose details may be found in [5] and references herein, the tRTE in its local form adapted to arbitrary refractive index distribution is obtained as

$$\begin{aligned}&\frac{n}{c}\frac{\partial I}{\partial t}+\nabla(\vec{\Omega}I)+\frac{1}{\sin\theta}\left\{\frac{\partial}{\partial\phi}\left[\frac{1}{n}\frac{\partial n}{\partial y}\cos\phi-\frac{1}{n}\frac{\partial n}{\partial x}\sin\phi\right]\right.\\ &\quad\left. I-\frac{\partial}{\partial\theta}\left[\frac{1}{n}\frac{\partial n}{\partial z}-\xi\left(\mu\frac{1}{n}\frac{\partial n}{\partial x}+\eta\frac{1}{n}\frac{\partial n}{\partial y}+\xi\frac{1}{n}\frac{\partial n}{\partial z}\right)\right]I\right\}\\ &\quad+(\kappa+\sigma_s)I-\frac{\sigma_s}{4\pi}\int_{4\pi} d\Omega' I\boldsymbol{\Phi}(\vec{\Omega}',\vec{\Omega})=0\end{aligned} \tag{4}$$