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Application of the lattice Boltzmann method to transient conduction and radiation heat transfer in cylindrical media

Raoudha Chaabane*, Faouzi Askri, Sassi Ben Nasrallah

Laboratoire d'Etudes des Systèmes Thermiques et Energétiques, Ecole Nationale d'Ingénieurs de Monastir, Avenue Ibn El Jazzar, 5019 Monastir, Tunisie

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ABSTRACT

In this paper, the lattice Boltzmann method (LBM) is applied to solve the energy equation of a transient conduction–radiation heat transfer problem in a two-dimensional cylindrical enclosure filled with an emitting, absorbing and scattering media. The control volume finite element method (CVFEM) is used to obtain the radiative information. To demonstrate the workability of the LBM in conjunction with the CVFEM to conduction–radiation problems in cylindrical media, the energy equation of the same problem is also solved using the finite difference method (FDM). The effects of different parameters, such as the grid size, the scattering albedo, the extinction coefficient and the conduction–radiation parameter on temperature distribution within the medium are studied. Results of the present work are compared with those available in the literature. LBM-CVFEM results are also compared with those given by the FDM-CVFEM. In all cases, good agreement has been obtained.

1. Introduction

The analysis of the transient conduction–radiation problem in cylindrical media finds applications in the design of reactors, heat pipes, combustion chambers, rocket propulsion systems, etc. The radiative information in the energy equation of a problem involving conduction and radiation can be obtained by various methods such as the Monte Carlo method (MCM) [1], the spherical harmonic method [2], the discrete transfer method (DTM) [3],the discrete ordinates method (DOM) [4], the finite-volume method (FVM) [5–7], the collapsed dimension method (CDM) [3] and the control volume finite element method (CVFEM) [8–11]. Among these methods, the CVFEM has been demonstrated to be successful in the solution of 2D [12] and 3D [13–15] rectangular enclosures, as well as for the unstructured mesh [16,17], and also for the solution of combined-mode heat transfer in participating media [18,19]. Grissa et al. [13,14] have proved its accuracy and computational efficiency in the case of the 3D enclosures. They demonstrate that this method is still observed to represent a very accurate result with reasonable CPU time and iterations and this highlights its important implications when coupling the RTE calculations with flow, conduction and convection heat transfer codes.

CVFEM is particularly a very promising approach for the solution of radiative transfer problems in cylindrical geometries [11]. The advantages of this method are: (i) it insures the flux conservation; and (ii) the used control volumes presented more faces (6 faces), which make it possible to avoid the numerical diffusion [8–10]. In addition, it was demonstrated in previous works [15] that the use of the CVFEM instead of other CFD methods (for example FVM or the cubic discrete ordinates interpolation method (DOIM)) reveals that false scattering occurs in all the tested methods but the CVFEM produces less errors. (iii) Six nodes are used for each calculation point; therefore, the stability of the numerical resolution process is improved. (iv)The control volume is treated as the addition

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* Corresponding author.
E-mail address: Raoudhach@gmail.com (R. Chaabane).

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List of symbols

$\vec{q}_R$	radiative heat flux (W m^{-2})
c_p	specific heat capacity (m^2 S^{-1} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
I	radiative intensity (W m^{-2} sr^{-1})
s	position (m)
k_a	absorption coefficient (m^{-1})
k_d	scattering coefficients (m^{-1})
$I_b(s)$	blackbody radiative intensity (W m^{-2} sr^{-1})
$\vec{n}_w$	unit normal vector on the wall
N_ϕ	numbers of control angles in the polar directions.
N_{θ_p}	numbers of control angles in the azimuthal directions
$\Delta\Omega^{mn}$	control solid angle
ΔV_{ik}	control volume
δV_{lik}	subvolume
N	node
G_i	centroids
r,z	direction
$\Delta r,\Delta z$	regular steps
L_r	the *r* dimensions of the calculation domain
L_z	the *z* dimensions of the calculation domain
A^N	surface of the control volume ΔV_{ik}
$\Phi^{mnm'n'}$	averaged scattering phase function
$\nabla\cdot\vec{q}_R$	radiative information
G	incident radiation (W m^{-2})
f_i	particle distribution function
$\vec{r}=(\vec{r}(r,z))$	lattice node
t	time (s)
i	direction
$\vec{c_i}$	velocity
$\Delta\vec{r}$	lattice link
$T(\vec{r},t)$	temperature (K)
$f_i^{(0)}(\vec{r},t)$	equilibrium function (K)
N	conduction–radiation parameter
Δt	time step, (s)

Greek symbols

Ω_i	collision operator
ξ	dimensionless time
β	extinction coefficient (m^{-1})
ω	scattering albedo
σ	Stefan–Boltzman constant (W m^{-2} K^{-4})
τ	relaxation time (s)
τ_R	optical thickness
ω_i	weights in D2Q9 lattice
ρ	density (Kg m^{-3})
$P(\overrightarrow{\Omega'}-\vec{\Omega})$	scattering phase function
$\vec{\Omega}$	outgoing direction of propagation
$\overrightarrow{\Omega'}$	incoming direction
ε_w	wall emissivity
θ	polar angle
ϕ	azimuthal angle

Subscripts

b	black body
ref	reference value

of six elements controls volumes that improved the grid flexibility.

Analyses of coupled conduction–radiation heat transfer in cylindrical media have also been reported by many researchers [20–23]. Chang and Smith [20] are the first to report investigations on combined conduction–radiation heat transfer in a cylindrical enclosure. They used the differential approximation to solve the radiative transfer equation, which along with the energy equation, was solved using the method of successive approximation. Fernandes and Francis [21] have applied the finite element method (FEM) to a conduction–radiation heat transfer problem in a concentric cylinder. Sakami et al. [22] and Dlala et al. [23] have studied 1-D conductive-radiative heat transfer problems in cylindrical media using the discrete ordinates method. Mishra et al. [24] applied the LBM–FVM to treat the transient conduction–radiation problem in a 1D cylindrical medium.

During the last decades, the lattice Boltzmann method (LBM) has found wide-ranging applications in science and engineering [25–27]. This surge in interest is mainly attributed to its ability, direct discretization, computational simplicity, ability and efficiency [25–27]. Unlike the conventional computational fluid dynamics (CFD) solvers, such as the finite difference method (FDM), the FEM and the FVM, which are based on macroscopic models, the LBM [26–28,32] is a mesoscopic approach and it describes and captures physics better [25,27]. This method includes simple calculations procedure, efficient implementation for a parallel architecture, simplicity of boundary condition's implementation, easy and robust handling of complex geometry and others [34–37]. The application of the LBM has gained momentum in the solution of transient conduction–radiation problems [24,29,33,38]. However, its application in cylindrical enclosures involving 2-D conduction–radiation heat transfer has not yet been reported. The aim of this work is to extend the application of the LBM in solving the energy equation of a 2-D transient conduction–radiation heat transfer problem in a cylindrical enclosure. The solution methodology adopted in this work involves the CVFEM [11] to solve the radiative transfer equation (RTE), followed by the LBM and the FDM for the solution of the energy equation. The results obtained from the LBM–CVFEM and the FDM–CVFEM combinations are compared for the effects of various parameters, such as the scattering albedo, the extinction coefficient and the conduction radiation parameter. Comparison of the number of iterations and the effect of grid size in the LBM–CVFEM and FDM–CVFEM combinations are also reported. Results of the present work are compared with those available in the literature, and in all cases, good agreement has been obtained.