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Analysis of radiative transport in a cylindrical enclosure—An application of the modified discrete ordinate method

Subhash C. Mishra*, Ch. Hari Krishna

Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Guwahati 781039, India

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ABSTRACT

Application of the modified discrete ordinate method (MDOM) proposed by Mishra et al. [*Mishra SC, Roy HK, Misra N. Discrete ordinate method with a new and simple quadrature scheme. J Quant Spectrosc Radiat Transfer 2006;101:249–262.*] has been extended for calculation of volumetric radiative information in a cylindrical enclosure. Radiatively, the medium inside a diffuse gray 1-D concentric cylinder is absorbing, emitting and scattering. Three types of problems, viz., an isothermal medium representing non-radiative equilibrium case, a non-isothermal medium representing radiative equilibrium situation and the case of a combined mode conduction and radiation heat transfer have been used to test the robustness of the MDOM. Temperature/emissive power and heat flux/energy flow rate distributions in the medium have been found for the effects of various parameters like the extinction coefficient, the scattering albedo, the boundary emissivity and the conduction–radiation parameter. To check the accuracy of the results of the MDOM, results have been compared with those available in the literature and also by obtaining the radiative information using the finite volume method. MDOM has been found to provide accurate results.

1. Introduction

In high temperature applications, thermal radiation is a dominant mode of heat transfer. Its consideration is of paramount importance in thermal characterization and design of high temperature systems such as boilers, furnaces, insulations, IC engines, gas turbine, building, tunnels, forest fires, etc. [1–7]. Its analysis is also important in materials processing involving phase change [8], characterization of a biological system [9,10] and in weather forecasting [11].

Transport of thermal radiation in a participating medium is a volumetric phenomenon. Apart from its dependence on spatial coordinates, although it is very fast process, in the transport of a short-pulse radiation, it has a temporal dependence [9,10]. Further, unlike conduction and convection, it depends on two angular dimensions, viz., polar and azimuthal angles, and a spectral dimension, i.e., wavelength. In many applications, the temporal dimension and its dependency on wavelength can either be neglected or they do not pose big problems in the solution. However, its dependence on the angular dimensions always needs to be accounted, and this accounting makes the analysis of radiative heat transfer not only difficult, but also computationally time consuming. At all computational nodes, in every iteration, in all discrete directions, intensities need to be traced. This is a time consuming process. The governing radiative transfer equation (RTE) turns out to be an integro-differential one.

Owing to the integro-differential nature of the RTE, since the start of research on thermal radiation for engineering applications in 1960, till date, analytic solution of the RTE has been reported only for simple geometry with simple boundary and medium conditions. Multi-dimensional geometries and difficult situations are still beyond the purview of analytic solutions. Further, the cases for which analytic solutions are available, their adaptability with the combined mode solver has not been so convenient and

* Corresponding author. Tel.: 91 361 2582660; fax: 91 361 2690762.
E-mail address: scm_iitg@yahoo.com (S.C. Mishra).

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Nomenclature

E, W, N, S	east, west, north and south nodes of the neighboring control volumes having node P, respectively
e, w, n, s	east, west, north and south face of the control volume having node P, respectively
$\vec{e}_r, \vec{e}_{\varphi_0}, \vec{e}_z$	cylindrical r-,φ_0- and z-direction base vectors, respectively
c_p	specific heat (J Kg^{-1} K^{-1})
G	incident radiation (W m^{-2})
I	intensity of radiation (W sr^{-1} m^{-2})
L	reference length (m)
$\vec{n}_i$	outward unit normal vector at face i
$\vec{n}_b$	unit normal vector at the wall towards the medium
N	conduction–radiation parameter, $\kappa\beta/4\sigma T_{ref}^3$
N_θ	number of angular divisions of the polar space
N_ϕ	number of angular divisions of the azimuthal space
P	present control volume
q	radiative heat flux (W m^{-2})
Q	energy flow rate (W)
r_1	radius of the inner cylinder (m)
r_2	radius of the outer cylinder (m)
S	source term (W m^{-3})
$\vec{\Omega}$	direction vector
T	temperature (K)
t	time (s)

Greek symbols

A, V	surface area and volume of a control volume, respectively
α	thermal diffusivity (m^2 s^{-1})
β	extinction coefficient (m^{-1})
ε	emissivity
ϕ	azimuthal angle (rad)
Φ	dimensionless emissive power
κ	thermal conductivity (W m^{-1} K^{-1})
Θ	dimensionless temperature
θ	polar angle (rad)
ϕ, ϕ_Ω	angular azimuthal angles measured from the r- and x-axis, respectively (rad) (Fig. 1b).
φ_0	spatial azimuthal angle measured from the x-axis (rad) (Fig. 1b)
κ_a	absorption coefficient (m^{-1})
σ	Stefan–Boltzmann constant (=5.670 × 10^{-8} W m^{-2} K^{-4})
σ_s	scattering coefficient (m^{-1})
μ, η	direction cosines in the r, and φ_o directions, respectively.
Ω	solid angle
$\Delta\Omega$	elemental solid angle (sr)
ω	scattering albedo
ξ	dimensionless time, $\alpha t/L^2$
Ψ	dimensionless heat flux

Subscripts

1, 2	inner and outer cylinder walls, respectively
b	black, boundary
C	conductive
E, W, N, S	control volume nodal points where intensities are located
e, w, n, s	control volume faces
P	value at the cell centre
R	radiative
ref	reference
T	total

Superscripts

*	dimensionless
m	discrete direction

Abbreviations

ADI	alternate direction implicit
ART	angular redistribution term
DOM	discrete ordinate method
DTM	discrete transfer method
FVM	finite volume method
LHS	left hand side
MDOM	modified discrete ordinate method
NCV	number of control volumes
RHS	right hand side
RTE	radiative transfer equation
SS	steady-state

thus the usage of the analytic solution in such problems is almost not in practice. Owing to these limitations, over the years, many numerical radiative transfer methods, such as the zonal method [12], the Monte Carlo method [13], the flux method [14], the discrete ordinates method (DOM) [15], spherical harmonics method [16], discrete transfer method [17], YIX method [18], the collapsed dimension method [19], the finite volume method (FVM) [20], lattice Boltzmann method [21], etc., with several of their modified versions have been developed. Each method has its strong and weak points, and some of them are more suited to particular applications. To simplify the formulation and for a better computational efficiency, the research is still ongoing to develop new methods and improve the exiting methods.

Although proposed in 1960 by Chandrashekhar for stellar applications [22] and being one of the oldest methods, the DOM with several of its modified versions continues to be one of the widely used radiative transfer methods in science and engineering [8–11,23–26]. It has found wide applications in varieties of problems [27]. In this, first the discrete directions are selected and the RTE is written for that direction. Then, for a