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Image reconstruction in diffuse optical tomography using the coupled radiative transport–diffusion model

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ABSTRACT

The coupled radiative transport–diffusion model can be used as light transport model in situations in which the diffusion equation is not a valid approximation everywhere in the domain. In the coupled model, light propagation is modelled with the radiative transport equation in sub-domains in which the approximations of the diffusion equation are not valid, such as within low-scattering regions, and the diffusion approximation is used elsewhere in the domain. In this paper, an image reconstruction method for diffuse optical tomography based on using the coupled radiative transport–diffusion model is developed. In the approach, absorption and scattering distributions are estimated by minimising a regularised least-squares error between the measured data and solution of the coupled model. The approach is tested with simulations. Reconstructions from different cases including domains with low-scattering regions are shown. The results show that the coupled radiative transport–diffusion model can be utilised in image reconstruction problem of diffuse optical tomography and that it produces as good quality reconstructions as the full radiative transport equation also in the presence of low-scattering regions.

1. Introduction

Diffuse optical tomography (DOT) is an imaging modality in which images of the optical properties of the medium are estimated based on measurements of near-infrared light on the surface of the object. It has potential applications in medical imaging, for example in breast cancer detection, monitoring of infant brain tissue oxygenation level and functional brain activation studies, for reviews see e.g. [1–3].

The image reconstruction problem in DOT is a non-linear ill-posed inverse problem. Thus, even small errors in the measurements or modelling can cause large errors in the reconstructions. To overcome the difficulties due to the ill-posed nature of the problem, regularisation techniques or Bayesian approach need to be used [1]. Furthermore, computationally feasible forward models that describe light propagation within the medium accurately are needed.

The radiative transport equation (RTE) is widely accepted as an accurate model for photon migration in tissues [1,4]. The RTE does not have analytical solutions for arbitrary geometries and the numerical solutions with sufficiently dense discretisations lead to computationally demanding problems. Therefore, the RTE has been used as a forward model only in few applications of DOT, see e.g. [5–12].

Due to the computationally intensive nature of the RTE problem, the typical approach in DOT has been to derive approximate, computationally less demanding models based on the RTE. The most typical approach has been to use the diffusion approximation (DA) to the RTE as the

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forward model. The DA is basically a special case of the first-order spherical harmonics approximation to the RTE, and thus it has some limitations. Firstly, the medium must be scattering dominated, and secondly, light propagation is not modelled accurately close to the collimated light sources and boundaries [1]. Due to these limitations, the DA fails to produce accurate estimates for light propagation in the proximity of the source and boundaries, and in cases in which the turbid medium contains low-scattering or non-scattering regions [13–15].

To overcome the limitations of diffusion theory, different hybrid models which combine diffusion equation with other models have been developed. These include the radiosity diffusion model [14,16] which can be utilised in highly scattering medium with non-scattering regions, hybrid Monte Carlo–diffusion approaches [13,17,15], and models in which the radiative transport equation is coupled with the diffusion equation [18–22] or different orders of P_N-approximation are applied [23]. However, except for the radiosity diffusion model [16], the inverse problem using these models has not been addressed.

In this paper, an image reconstruction method based on using the coupled radiative transport–diffusion model [18,20–22] is developed for the inverse problem of DOT. The main aim of the paper is to demonstrate that the coupled model provides a viable calculation to be employed within an inverse transport reconstruction. The coupled model was developed as a light propagation model for DOT in Bal and Maday [18] and Tarvainen et al. [20]. It was extended for low-scattering regions in Tarvainen et al. [21] and for three-dimensional fluorescence imaging in Gorpas et al. [22]. In the coupled model, light propagation is modelled with the radiative transport equation in sub-domains in which the approximations of the diffusion equation are not valid and the diffusion approximation is used elsewhere in the domain. In the inverse problem of DOT, the absorption and scattering distributions within the object are estimated by minimising a regularised least-squares error between the measured data and data obtained as a solution of the coupled model. In this work, this minimisation problem is solved with Gauss–Newton method which is equipped with a line search algorithm and a positivity constraint for the estimated parameters. In addition, scaling of the data and solution spaces is applied in order to improve the performance of the optimisation algorithm.

The rest of the paper is organised as follows. The forward problem of DOT and light transport models are reviewed in Section 2 and the inverse problem is described in Section 3. The finite element (FE) implementations are given in Section 4. The simulation results are shown in Section 5 and conclusions are given in Section 6.

2. Forward problem

The forward problem in DOT is to compute the measurable data when the optical properties of the medium and the input light sources are given. Light propagation in biological media is usually modelled through transport theory which can be treated through stochastic methods, such as Monte Carlo, or deterministic methods which are based on describing particle transport with integro-differential equations. In the following we consider the latter approach.

2.1. Radiative transport equation

A widely accepted model for light transport in tissues is the radiative transport equation [24]. The RTE is a one-speed approximation of the transport equation, and thus it assumes that the energy (or speed) of the photons does not change in collisions and that the refractive index is constant within the medium.

Let $\Omega \subset \mathbb{R}^n, n=2$ or 3 denote the physical domain with boundary $\partial\Omega$ and let $\hat{s} \in S^{n-1}$ denote a unit vector in the direction of interest. The RTE is written in the frequency domain as

$$\frac{\mathrm{i}\omega}{c}\phi(r,\hat{s})+\hat{s}\cdot\nabla\phi(r,\hat{s})+(\mu_s+\mu_a)\phi(r,\hat{s}) = \mu_s\int_{S^{n-1}}\Theta(\hat{s}\cdot\hat{s}')\phi(r,\hat{s}')\mathrm{d}\hat{s}'+q(r,\hat{s}), \quad r\in\Omega \tag{1}$$

where c is the speed of light in medium, i is the imaginary unit, ω is the angular modulation frequency of the input signal, and $\mu_s=\mu_s(r)$ and $\mu_a=\mu_a(r)$ are the scattering and absorption coefficients of the medium, respectively. Further, $\phi(r,\hat{s})$ is the radiance, $\Theta(\hat{s}\cdot\hat{s}')$ is the scattering phase function, and $q(r,\hat{s})$ is the source inside Ω. In this paper, there are no internal light sources, and thus $q(r,\hat{s})=0$.

The scattering phase function $\Theta(\hat{s}\cdot\hat{s}')$ describes the probability that a photon with an initial direction $\hat{s}'$ will have a direction $\hat{s}$ after a scattering event. In DOT, the most commonly used phase function is the Henyey–Greenstein scattering function [25] which is of the form

$$\Theta(\hat{s}\cdot\hat{s}')=\begin{cases}\dfrac{1}{2\pi}\dfrac{1-g^2}{(1+g^2-2g\hat{s}\cdot\hat{s}')}, & n=2\\[2ex] \dfrac{1}{4\pi}\dfrac{1-g^2}{(1+g^2-2g\hat{s}\cdot\hat{s}')^{3/2}}, & n=3\end{cases} \tag{2}$$

The scattering shape parameter g defines the shape of the probability density and it gets values between $-1<g<1$. With the value $g=0$, the scattering probability density is a uniform distribution. For forward dominated scattering $g>0$ and for backward dominated scattering $g<0$.

In DOT, we use the RTE boundary condition which assumes that no photons travel in an inward direction at the boundary $\partial\Omega$ except at source position $\varepsilon_j \subset \partial\Omega$, thus

$$\phi(r,\hat{s})=\begin{cases}\phi_0(r,\hat{s}), & r\in\varepsilon_j, \quad \hat{s}\cdot\hat{n}<0\\ 0, & r\in\partial\Omega\backslash\varepsilon_j, \quad \hat{s}\cdot\hat{n}<0.\end{cases} \tag{3}$$

where $\hat{n}$ is an outward unit normal and $\phi_0(r,\hat{s})$ is a boundary source [1,26]. This boundary condition implies that once a photon escapes the domain Ω it does not reenter it.

In DOT, the measurable quantity is the excitance $J^+(r)$ on the boundary of the domain. Utilising the boundary