Journal of Quantitative Spectroscopy & Radiative Transfer 111 (2010) 742–752

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Phase function of long helical particles at normal and oblique incidence

Ioseph Gurwich [b], Moshe Kleiman [a,*], Nir Shiloah [a]

[a] Israel Institute for Biological Research, P.O. Box 19, Ness-Ziona 74100, Israel
[b] ELOP, P.O. Box 1166, Rehovot 76110, Israel

ARTICLE INFO

Article history:
Received 6 September 2009
Accepted 30 October 2009

Keywords:
Light scattering
Long helix
Hollow cylinder
Nonspherical particles

ABSTRACT

This paper studies characteristics of the electro-magnetic wave scattering by a long helical particle in cases of oblique incidence with different polarizations. The dependence of the resonant peaks and the phase function on the incidence angle is analyzed for different values of helix parameters. The analysis is based on the model developed by the authors and presented in previous publications.

1. Introduction

The possibility of treating the problem of electromagnetic scattering by a long helical particle by using a Fourier formalism was demonstrated recently [1]. Actually this method can be considered as an extension of the Mie approach. The suggested formalism is based on the representation of a helical particle as a thin non-homogeneous membrane with periodical boundary conditions. It has been shown that in this case one can use a calculation procedure developed for a multi-layered (hollow) cylinder [2] in order to find all Fourier (diffraction) orders of the scattered field. The algorithm and the computer code developed in [1] were verified by comparison to the theoretical calculations performed by two other research groups and to the experimental data on microwave scattering by a helix.

Few numerical examples presented in [1] show the resonant behavior of scattering by a long helix at sufficiently high frequencies. It was shown how this resonant scattering is affected by helix parameters (pitch Λ and the central radius R_c). In the present work we generalize the analysis for studying some properties of scattering of electromagnetic radiation by a long helical particle. In particular, we investigate here the phase function and polarization of scattered radiation for normal and oblique incidence and for different values of helix parameters. We also analyze the relation between diffraction peaks and frequency-dependence of different Mie coefficients. The symmetry properties of the incidence angle dependence and the phase function are considered. All the numerical examples involve right helixes.

2. Oblique incidence

Studying the frequency dependence of the scattering intensity by normal incidence, we saw [1] that resonance peaks appear when the wavelength of the incident wave becomes smaller than the helix period ($\lambda < \Lambda$). This corresponds to the general condition (grating equation), under which a grating structure exhibits its diffractive ability. Here we intend to examine the dependence of resonance frequency on the incidence angle and effect of this angle on the phase function. One additional effect we study here is depolarization. For a more detailed picture,

* Corresponding author. Tel.: +972 89381654; fax: +972 89381664.
E-mail address: moshekl@iibr.gov.il (M. Kleiman).

0022-4073/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jqsrt.2009.10.022

we also analyze separate diffraction orders ($Q_{sc,p}$) as well as the total field ($Q_{sc,T}$).

$$Q_{sc,p} = \sum_n (|A_{n,p}|^2 + |B_{n,p}|^2)$$

$$Q_{sc,T} = \sum_n \sum_p (|A_{n,p}|^2 + |B_{n,p}|^2) \tag{1}$$

Here $A_{n,p}$ and $B_{n,p}$ are coefficients in the Fourier expansion of Mie scattering coefficients a_n and b_n [1] as

$$a_n(z,\phi) = \sum_p A_{n,p} e^{ip\phi} e^{i2\pi lzp\Lambda}$$

$$b_n(z,\phi) = \sum_p B_{n,p} e^{ip\phi} e^{i2\pi pz/\Lambda} \tag{2}$$

where ϕ is the polar angle and Λ is the pitch of the helix.

This helps in understanding the origin of different peaks (as shown in Fig. 1) and reconstructing depolarization processes.

In the figure $\mathbf{K} = (2\pi/\Lambda)\hat{\mathbf{z}}$ is the helix "wave vector", and $\mathbf{k}_{+n}=\mathbf{k}_{in}+\mathbf{K}$ and $\mathbf{k}_{-n}=\mathbf{k}_{in}-\mathbf{K}$ are the wave vectors the proper diffraction orders.

First of all, one can expect that in oblique incidence the resonant diffraction peaks will be sharper. Here an analogy with Bragg diffraction can be recognized (see also a brief discussion in [4]), though here there is not an exact Bragg condition; the cylindrical geometry prescribes also the polar angular dependence, i.e., different "optimal conditions" for scattering in different polar angles (as seen from the analysis of the phase function). The change of the first peak position and shape with varying incident angle for the total scattered energy is shown in Fig. 2 (in this and following graphs and calculations the incident angle α is taken with respect to the helix axis, thus α=90° corresponds to normal incidence). In this example the incident wave has a polarization vector **E** parallel to the incident plane.

We really see much sharper and higher peaks, when the incidence is not normal. At the same time we also see the increase of the zero order scattering (i.e., the scattering from an equivalent cylinder). For large enough incidence angles the resonant peak becomes low and gradually non-recognizable on the increasing zero-order background. The shift of the "peak frequency" is determined by the "effective periodicity", changing with the incidence angle, or in other words, by the phase condition, analogous to the well-known grating equation. Thus, while for the normal incidence the diffraction orders appear with $\lambda < \Lambda$, the proper condition for oblique incidence is

$$\lambda < \frac{\Lambda}{1+\cos\alpha} \tag{3}$$

For more general comparison of the scattering characteristics we present the frequency dependence of the total scattering intensity for normal incidence and α=45° in a wide frequency range (Fig. 3).

One can note that not only the main peaks but also the whole characteristic become sharper with the passage from the normal to oblique illumination.

The following Fig. 4 details the appearance of different diffraction orders for the same helix by 45°-incidence and the correspondence of the peaks of the total scattering characteristic to those of different diffraction orders.

Another aspect, where one has to consider different orders separately, is the analysis of the polarization of the scattered field. Since a certain (pth) order is a cylindrical wave (a superposition of cylindrical harmonics), we can analyze its polarization by using the proper Mie coefficients (see [3]), actually their Fourier components $A_{n,p}$ and $B_{n,p}$, as

- for an input wave polarized in the incident plane (||)

$$E_{s,\parallel}^{(p)}(\phi) = B_{0,p}\cos\{p\phi\} + 2\sum_n B_{n,p}\cos\{(n\pm p)\phi\}$$

$$E_{s,\perp}^{(p)}(\phi) = -iA_{0,p}\sin\{p\phi\} - 2i\sum_n A_{n,p}\sin\{(n\pm p)\phi\} \tag{4a}$$

- for an input wave polarized normal to the incident plane (⊥)

$$E_{s,\parallel}^{(p)}(\phi) = -iB_{0,p}\sin\{p\phi\} - 2i\sum_n B_{n,p}\sin\{(n\pm p)\phi\}$$

$$E_{s,\perp}^{(p)}(\phi) = A_{0,p}\cos\{p\phi\} + 2\sum_n A_{n,p}\cos\{(n\pm p)\phi\} \tag{4b}$$

where the sign $\pm$ corresponds to a right or left helix, respectively.

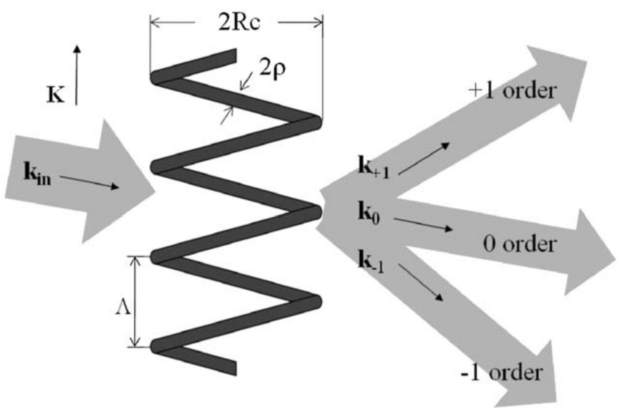

Fig. 1. The scattered field of a separate diffraction orders. The total scattered field is a coherent summation of all diffraction orders.