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Stability in Debye series calculation for light scattering by absorbing particles and bubbles

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ABSTRACT

The Debye-series decomposition is of importance for understanding of light scattering features and for the validity of the geometrical optics approximation to light scattering. The numerical stability and accuracy for calculating light scattering with Debye series is studied and an improved algorithm is proposed in this work. The ratios of the Riccati–Bessel functions and the logarithmic derivatives of the Riccati–Bessel functions are employed and calculated with proper recurrences. Exemplifying results are provided to show the improvement of the algorithm.

1. Introduction

The Lorenz-Mie theory [1–3], expressed in terms of an infinite series, provides a rigorous solution to the problem of light scattering from homogeneous spheres. However, it gives a global description of the scattering but does not give much insight into the physical processes involved in scattering such as the multiple reflections and refractions at the interfaces of the scatterer. “It turns out that writing each term of the Mie infinite series as another infinite series, known as the Debye series, clarifies the physical origins of many effects that occur in electromagnetic scattering” [4,5]. The Debye series expansion (DSE) allows for the decomposition of the global physical process in a series of local interactions. The reflections and the refractions at each interface of the scatterer are therefore clearly shown. In this sense this method brings a better physical understanding. The use of Debye series has been a key to the understanding of light scattering features, such as rainbows, glories, coronas, and other phenomena [6–14]. Furthermore, it is also used as a criterion to validate the geometrical optics approximation of light scattering [15–17]. Nevertheless, numerical calculation of the Debye series is much more difficult than the calculation of the Mie series. Some computational problems are still left unsolved in the Debye calculation. This paper presents an improved algorithm for the calculation of the Debye series. The ratios of the Riccati–Bessel functions and the logarithmic derivatives of the Riccati–Bessel functions are employed and are calculated with the different recurrences. Numerical calculations are tested to check the stability and the accuracy of the algorithm, by comparing the numerical results of the Debye series calculation to those of the Mie calculation.

2. Mie coefficients and Debye series

The light scattering of a plane wave by a homogeneous sphere, given in the Mie series, is expressed in terms of the Mie scattering coefficients a_n and b_n, which can be calculated with [18,19]

$$a_n = A_n^{(1l)}(x)\frac{D_n^{(1)}(y) - mD_n^{(1)}(x)}{D_n^{(1)}(y) - mD_n^{(l)}(x)}$$

$$b_n = A_n^{(1l)}(x)\frac{mD_n^{(1)}(y) - D_n^{(1)}(x)}{mD_n^{(1)}(y) - D_n^{(l)}(x)} \quad (1)$$

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where $D_n^{(l)}(z)$ is the logarithmic derivative of the Riccati–Bessel functions, defined as

$$D_n^{(j)}(z) = \frac{d}{dz}\ln R_n^{(j)}(z) \tag{2}$$

l is taken to be 3 or 4 depending on the choice of the time convention $\exp(-i\omega t)$ or $\exp(i\omega t)$. The Riccati–Bessel functions are $R_n^{(1)}(z)=\psi_n(z)$, $R_n^{(2)}(z)=\chi_n(z)$, $R_n^{(3)}(z)=\xi_n^{(1)}(z)=\psi_n(z)-i\chi_n(z)$ and $R_n^{(4)}(z)=\xi_n^{(2)}(z)=\psi_n(z)+i\chi_n(z)$, respectively. The variable z is the dimensionless particle size parameter x(i.e. $z=x=\pi d/\lambda$, where d is the particle diameter and λ the wavelength of the light in vacuum) or the product of the particle size parameter x and the relative refractive index m (i.e. $z=y=mx$). The relative refractive index $m=m_1/m_2$ is the ratio of the refractive index of the sphere m_1 to that of the medium m_2. For an absorbing particle, the relative refractive index m is complex and its imaginary part may be positive or negative depending on the choice of the time convention. $A_n^{(kl)}(z)$ is the ratio of the Riccati–Bessel functions, defined as

$$A_n^{(kl)}(z) = \frac{R_n^{(k)}(z)}{R_n^{(l)}(z)} \tag{3}$$

The calculation of the Mie coefficients a_n and b_n requires the calculation of the logarithmic derivatives of the Riccati–Bessel functions (i.e. $D_n^{(1)}(x)$, $D_n^{(1)}(y)$ and $D_n^{(l)}(x)$) and the ratio of the Riccati–Bessel functions $A_n^{(1l)}(x)$.

As long as the scattering coefficients a_n and b_n are calculable, physical quantities in light scattering such as extinction efficiency k_{ext}, scattering efficiency k_{sca}, absorbing efficiency k_{abs}, the complex scattering amplitudes for two orthogonal directions of incident polarization s_1 and s_2 as well as the scattering intensities i_1 and i_2 can be calculated [1–3]:

$$k_{\text{ext}} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\text{Re}(a_n+b_n)$$

$$k_{\text{sca}} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)(|a_n|^2+|b_n|^2)$$

$$k_{\text{abs}} = k_{\text{ext}} - k_{\text{sca}} \tag{4}$$

$$i_1(\theta) = |s_1(\theta)|^2 = \left|\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}[a_n\pi_n(\theta)+b_n\tau_n(\theta)]\right|^2$$

$$i_2(\theta) = |s_2(\theta)|^2 = \left|\sum_{n=1}^{\infty}\frac{2n+1}{n(n+1)}[a_n\tau_n(\theta)+b_n\pi_n(\theta)]\right|^2$$

$$i(\theta) = \frac{i_1(\theta)+i_2(\theta)}{2} \tag{5}$$

where $\pi_n(\theta)$ and $\tau_n(\theta)$ are the angular functions, which can be expressed in terms of the associated Legendre polynomials and can be computed by upward recurrence from the relations [1]

$$\pi_n(\theta) = \frac{2n-1}{n-1}\cos\theta\,\pi_{n-1}(\theta) - \frac{n}{n-1}\pi_{n-2}(\theta)$$

$$\tau_n(\theta) = n\cos\theta\,\pi_n(\theta) - (n+1)\pi_{n-1}(\theta) \tag{6}$$

beginning with π_0=0 and $\pi_1=1$.

The scattering coefficients a_n and b_n can also be written as a decomposition of Debye-series so that they can represent the separated contributions of the scattered waves [4,9].

$$a_n = \frac{1}{2}\left[1 - R_{n,\text{TM}}^{212} - \frac{T_{n,\text{TM}}^{21}T_{n,\text{TM}}^{12}}{1-R_{n,\text{TM}}^{121}}\right] = \frac{1}{2}\left[1 - R_{n,\text{TM}}^{212} - \sum_{p=1}^{\infty}T_{n,\text{TM}}^{21}(R_{n,\text{TM}}^{121})^{p-1}T_{n,\text{TM}}^{12}\right]$$

$$b_n = \frac{1}{2}\left[1 - R_{n,\text{TE}}^{212} - \frac{T_{n,\text{TE}}^{21}T_{n,\text{TE}}^{12}}{1-R_{n,\text{TE}}^{121}}\right] = \frac{1}{2}\left[1 - R_{n,\text{TE}}^{212} - \sum_{p=1}^{\infty}T_{n,\text{TE}}^{21}(R_{n,\text{TE}}^{121})^{p-1}T_{n,\text{TE}}^{12}\right] \tag{7}$$

The subscripts TE and TM denote the polarization in the light scattering. R_n^{212} and R_n^{121} are the partial-wave reflection coefficients, T_n^{12} and T_n^{21} are the partial-wave transmission coefficients (as denoted in Fig. 1). By using the ratio of the Riccati–Bessel functions and the logarithmic derivatives of the Riccati–Bessel functions, they are expressed as [9]

$$T_{n,\text{TE}} = m\frac{A_n^{(34)}(y)}{A_n^{(34)}(x)}\frac{D_n^{(3)}(x)-D_n^{(4)}(x)}{D_n^{(3)}(x)-mD_n^{(4)}(y)}\frac{D_n^{(3)}(y)-D_n^{(4)}(y)}{D_n^{(3)}(x)-mD_n^{(4)}(y)}$$

$$R_{n,\text{TE}}^{212} = -\frac{1}{A_n^{(34)}(x)}\frac{D_n^{(4)}(x)-mD_n^{(4)}(y)}{D_n^{(3)}(x)-mD_n^{(4)}(y)}$$

$$R_{n,\text{TE}}^{121} = -A_n^{(34)}(y)\frac{D_n^{(3)}(x)-mD_n^{(3)}(y)}{D_n^{(3)}(x)-mD_n^{(4)}(y)} \tag{8}$$

$$T_{n,\text{TM}} = m\frac{A_n^{(34)}(y)}{A_n^{(34)}(x)}\frac{D_n^{(3)}(x)-D_n^{(4)}(x)}{mD_n^{(3)}(x)-D_n^{(4)}(y)}\frac{D_n^{(3)}(y)-D_n^{(4)}(y)}{mD_n^{(3)}(x)-D_n^{(4)}(y)}$$

$$R_{n,\text{TM}}^{212} = -\frac{1}{A_n^{(34)}(x)}\frac{mD_n^{(4)}(x)-D_n^{(4)}(y)}{mD_n^{(3)}(x)-D_n^{(4)}(y)}$$

$$R_{n,\text{TM}}^{121} = -A_n^{(34)}(y)\frac{mD_n^{(3)}(x)-D_n^{(3)}(y)}{mD_n^{(3)}(x)-D_n^{(4)}(y)} \tag{9}$$

where $T_{n,\text{TE}} = T_{n,\text{TE}}^{21}T_{n,\text{TE}}^{12}$ and $T_{n,\text{TM}} = T_{n,\text{TM}}^{21}T_{n,\text{TM}}^{12}$.

Each term of the right-hand side of Eq. (7) has a clear physical interpretation. The first term describes the

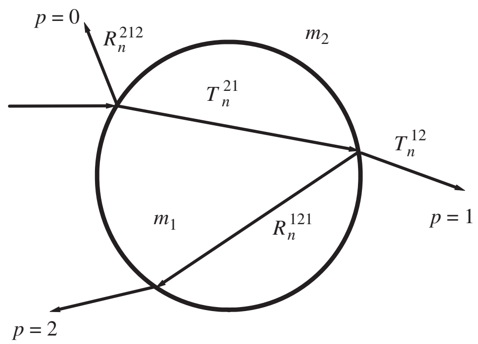

Fig. 1. Schematic of the Debye series.