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Explanation of the patterns in Mie theory

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ABSTRACT

The far-field scattered light intensity, or the related phase function, for a spherical particle is known to display an overall power-law structure when formulated in terms of the scattering wave vector. Empirically determined patterns in the intensity relating to the particle size and refractive index are known. The cause of the patterns, however, has not been satisfactorily explained. This work applies an exact microphysical model to explain most of the patterns, and specifically, to reveal the physical cause of crossovers from one power-law to another. A unique aspect of this microphysical approach is phasor analysis, which provides a visually based way to examine the angle-dependent wavelet superposition involved in the model. A simple color coding scheme connects the phasors to the interior of the particle, and it is this connection that reveals the meaning of the crossovers.

1. Introduction

In the far-field zone of a uniform spherical particle, the scattered light intensity provided by Mie theory is known to display an overall power-law structure [1,2]. This power-law structure is seen when the intensity distribution is formulated in terms of the scattering wave vector **q**, rather than in terms of the polar and azimuthal angles. Patterns appear in the power-law structure as the particle radius R and refractive index m are varied. An *empirically based* study of these patterns reveals simple relationships between R and the real part of m and characteristic features in the far-field angular scattered intensity distribution [1]. These empirical relationships make the patterns useful in situations where one desires a simple method to identify estimates for R and $\mathrm{Re}\{m\}$ directly from the scattered intensity, or the related phase function. Moreover, the simplicity of the patterns allows one to semi-quantitatively describe the form of the angular intensity for *any* spherical particle without recourse to sophisticated numerical calculation. Since their discovery, the patterns have received attention in both theoretical and experimental contexts [3–11]. Examples of laboratory measurements displaying the patterns can be found in [1,5,9,12].

In principle, the explanation for the occurrence and quantitative form of the power-law patterns is contained in the Mie solution. Unfortunately though, the mathematical complexity of the solution obscures a clear identification of their cause and physical significance. Consequently, simple models have been presented that explain the patterns using scaling arguments, Huygens' principle, and ray-tracing [1,8]. While these models can account for much of the patterns, certain inherent assumptions prevent them from being convincing and general explanations.

The purpose of this work is to apply an exact microphysical model to explain the origin of the patterns and the physical meaning of their various indicative features. The following will first present a short review of the q-space concept, the patterns, and the microphysical

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model. The model will then be applied to the problem of scattering by a single spherical particle in the weak and strong refraction regimes. These regimes are quantified by when the particle's phase shift parameter ρ, Eq. (4), is less than one and greater than one, respectively. A graphically based technique describing the scattering process, called phasor analysis, will be applied to both the $\rho<1$ and $\rho>1$ cases. Although a considerable amount of analysis will be required, the key points of this work will be to show that:

- The power-law form of the scattered intensity is related to the curvature of the particle's surface.
- Transitions between power-law regimes, called crossovers, are caused by the onset of destructive interference over specific regions within the particle.
- Characteristic length scales of the particle are associated with the crossovers.

All in all, this work will provide one with a conceptual interference-based understanding for the cause of the overall form of a spherical particle's far-field scattered intensity distribution.

2. Background: *q*-space and the patterns

Consider a spherical particle of volume V_{int} located at the origin $\mathcal{O}$ of the coordinate system. For simplicity, the particle is assumed to reside in vacuum and have only real-valued refractive index m. Illuminating the particle is a linearly polarized electromagnetic plane wave traveling in the $\hat{\mathbf{n}}^{inc}$ direction. The electric field of this incident wave is given by

$$\mathbf{E}^{inc}(\mathbf{r}) = \mathbf{E}_o^{inc}\exp(ikr\hat{\mathbf{r}}\cdot\hat{\mathbf{n}}^{inc}), \quad (1)$$

where $\mathbf{E}_o^{inc}$ is a constant vector describing the amplitude and polarization of the wave and k is the wavenumber given by $k=2\pi/\lambda$ where λ is the vacuum wavelength. All field quantities in this work are assumed to be time-harmonic and described by the factor $\exp(-i\omega t)$, where $\omega=kc$, with c being the speed of light. This global time factor will be suppressed for brevity. A detector is located at the observation point $\mathbf{r}$, which resides in the horizontal scattering plane. This plane is perpendicular to the polarization of the incident field. It will be convenient in the following to take $\mathbf{E}_o^{inc}=E_o^{inc}\hat{\mathbf{x}}$, where E_o^{inc} is a real-valued constant and $\hat{\mathbf{n}}^{inc}=\hat{\mathbf{z}}$. Then, the detector resides in the $y-z$ plane through the origin, see Fig. 1. Also, the detector is assumed to remain at a fixed radial distance R_l from the particle, i.e., $\mathbf{r}=R_l\hat{\mathbf{r}}$, where the size of the constant R_l is such that $\mathbf{r}$ resides in the particle's far-field zone following [13].

The power-law patterns appear when the far-field scattered intensity I is plotted log–log in terms of the scattering wave vector $\mathbf{q}$, rather than in terms of the polar scattering angle θ. This vector represents the difference in momentum between the incident plane wave and the far-field scattered wave in the $\hat{\mathbf{r}}$ direction, see Fig. 1 [22],

$$\mathbf{q}=k(\hat{\mathbf{n}}^{inc}-\hat{\mathbf{r}}). \quad (2)$$

The magnitude of $\mathbf{q}$ depends on θ as

$$q(\theta)=2k\sin\left(\frac{\theta}{2}\right), \quad (3)$$

and has the units of inverse length. Note that $\mathbf{q}$ lies in the same scattering plane as $\mathbf{r}$. In the following, the dependence of q on θ will be tacitly assumed and suppressed for brevity.

The semi-log plot (a) in Fig. 2 shows the evolution of the normalized intensity for spherical particles with various size parameters kR. These curves are generated from Mie theory following [20] using a refractive index of $m=1.05$. Plot (b) shows the same intensity curves, except plotted log–log as a function of the unitless quantity qR. One can see from (b) that all of the intensity curves are roughly bounded by linear envelopes, which given the log–log scale, indicates an overall power-law dependence of I on qR. In a sense, one can regard the envelopes as a coarse average of the more detailed ripple structure. In contrast, notice that the decay of the curves in (a) with angle does not clearly display a *specific* power-law form nor a transition from one power-law to another. The term

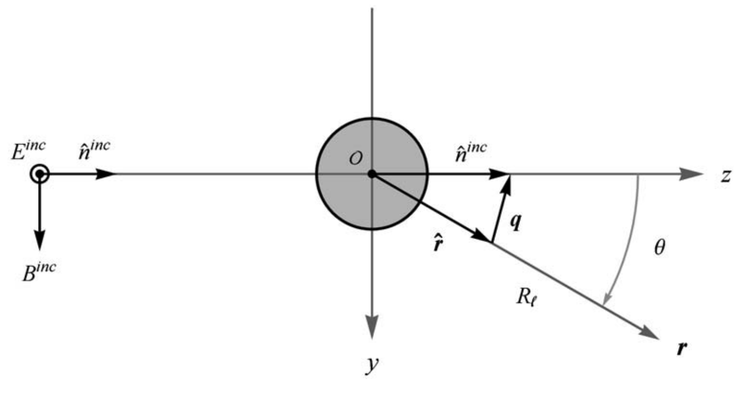

Fig. 1. Scattering arrangement consisting of a spherical particle illuminated by a linearly polarized plane wave. The observation point $\mathbf{r}$ is confined to the $y-z$ plane at a fixed radial distance R_l from the particle. The polarization of the incident wave is normal to $y-z$ plane. Also indicated is the direction of the scattering wave vector $\mathbf{q}$, Eq. (2).