Journal of Quantitative Spectroscopy & Radiative Transfer 111 (2010) 1015–1024

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Remarks on the radiative transfer approach to scattering of electromagnetic waves in layered random media

Saba Mudaliar *

Air Force Research Laboratory, Sensors Directorate, Hanscom AFB, MA 01731-2909, USA

ARTICLE INFO

Article history:
Received 23 August 2009
Received in revised form
30 November 2009
Accepted 1 December 2009

Keywords:
Random media
Rough surfaces
Radiative transfer
Multilayer
Wave approach

ABSTRACT

The radiative transfer (RT) approach is widely used in applications involving scattering from layered random media with rough interfaces. Although it has been successful in several applications in various disciplines it is well known that this approach involves certain approximations. In this paper these assumptions and approximations are reexamined. To enable this a statistical wave approach is employed to this problem and the governing equations for the first and second moments of the wave functions are derived. A transition is hence made to arrive at a system of equations corresponding to that of the RT approach. It is hence found that more conditions are implicitly involved in the RT approach than generally believed to be sufficient.

1. Introduction

The model of layered random medium with rough interfaces is often encountered in many applications in various disciplines. A simple approach is to incoherently add the contributions of volumetric and surface fluctuations [1,2]. However, this is valid only when we are in the single scattering regime [3,4]. There are some other hybrid approaches [5,6] which take into consideration some multiple scattering effects. Brown [7] outlines an iterative procedure which properly includes all multiple scattering interactions. However, it does not appear feasible to carry out the calculation beyond one or two iterations. Among the other methods currently used, perhaps the most widely used approach is the radiative transfer (RT) approach [8–15]. Here one formulates the scattering and propagation in each layer by using the radiative transfer equation which involves only the parameters of the medium of that layer. The boundary conditions are derived separately and independently using some asymptotic procedure developed in rough surface scattering theory [16–18]. The RT equations, along with the boundary conditions, comprise the system that describes the problem.

Although this procedure appears to be reasonable and sound it is apparent that certain approximations are involved and we would like to know the conditions under which this kind of approach is appropriate. One way to better understand the RT approach is to compare it with the more rigorous wave approach. For the case of unbounded random media it was found [19] that the RT approach is applicable when (a) one uses the quasi-uniform field approximation, (b) one uses the ladder approximation to the intensity operator of the Bethe–Salpeter equation, and (c) the medium is statistically quasi-homogeneous.

However, our problem has bounded structures which are randomly rough. Therefore it remains to be seen whether the conditions arrived at in the case of unbounded random media will be sufficient for our problem.

In this paper we employ a statistical wave approach using surface scattering operators [18,20] to derive the transport equations for our multilayer problem. In this process we find that there are more conditions implied

* Tel.: +1 781 377 2986.
E-mail address: saba.mudaliar@hanscom.af.mil

0022-4073/$ - see front matter Published by Elsevier Ltd.
doi:10.1016/j.jqsrt.2009.12.001

when we choose to apply the RT approach to our problem than it is widely believed to be necessary. One such condition is the weak surface correlation approximation. This means that the RT approach places certain restrictions on the type of rough interfaces that it can model accurately.

The paper is organized as follows. In Section 2 we describe the geometry of the problem. In the next section we describe the RT approach to the problem. In Section 4 we describe the wave approach to this problem. In Section 5 we transition to the transport equation system. The paper concludes with a discussion of our findings.

2. Description of the problem

The geometry of the problem is shown in Fig. 1. We have an N-layer random medium stack with rough interfaces. The permittivity of the j-th layer is $\varepsilon_j+\tilde{\varepsilon}_j(\mathbf{r})$ where ε_j is the deterministic part and $\tilde{\varepsilon}_j$ is the randomly fluctuating part. The permeability of all the layers is that of free space. The randomly rough interfaces are given as $z=z_j+\zeta_j(\mathbf{r}_\perp)$. It is assumed that $\tilde{\varepsilon}_j$ and ζ_j are zero-mean isotropic stationary random processes independent of each other. Thus, on the average the interfaces are parallel planes. Let $z_0=0$, and let d_j be the thickness of the j-th layer. The media above and below the stack are homogeneous with parameters ε_0, k_0, and ε_{N+1}, k_{N+1}, respectively. This system is excited by a monochromatic electromagnetic plane wave and we are interested in formulating the resulting multiple scattering process.

3. Radiative transfer approach

Multiple scattering in a complex environment is well described by the radiative transfer theory. This theory is not only conceptually simple but also very efficient. The fundamental quantity here is the specific intensity $\mathbf{I}$ which is governed by the following equation [21–23]:

$$\hat{s}\cdot\nabla\mathbf{I}(\mathbf{r},\hat{s})+\overline{\overline{\gamma}}\,\mathbf{I}(\mathbf{r},\hat{s})=\int d\Omega'\,\overline{\mathbf{P}}(\hat{s},\hat{s}')\mathbf{I}(\mathbf{r},\hat{s}'). \tag{1}$$

One may regard this equation as a statement of conservation of energy density $\mathbf{I}$ which is a phase-space quantity at position $\mathbf{r}$ and direction $\hat{s}$. $\overline{\overline{\gamma}}$ is the extinction matrix which is a measure of loss of energy in direction $\hat{s}$ due to scattering in other directions. $\overline{\mathbf{P}}$ is the phase matrix representing increase in energy density in direction $\hat{s}$ due to scattering from neighbouring elements. Ω is the solid angle subtended by $\hat{s}$. Given the statistical characteristics of the medium one can readily calculate the phase matrix. The extinction matrix is hence calculated using the relation $\overline{\overline{\gamma}}=\int\overline{\mathbf{P}}(\hat{s}',\hat{s})\,d\Omega'$. The specific intensity in each layer is governed by an equation similar to (1). Since our layer problem has translational invariance in azimuth the RT equation for the m-th layer takes the following form:

$$\cos\theta\frac{d}{dz}\mathbf{I}_m(z,\hat{s})+\overline{\overline{\gamma}}_m\mathbf{I}_m(z,\hat{s})=\int_{\Omega_m}d\Omega'\,\overline{\mathbf{P}}_m(\hat{s},\hat{s}')\mathbf{I}_m(z,\hat{s}'), \tag{2}$$

where the subscript m denotes that the quantity corresponds to those of the m-th layer and θ is the elevation angle of $\hat{s}$. This set of RT equations is complemented by a set of boundary conditions which are in turn based on energy conservation considerations. To be more precise, we impose the condition that the energy flux density at each interface is conserved. This leads to the following boundary conditions on the m-th interface:

$$\mathbf{I}_m^u(z_m,\hat{s})=\int d\Omega'\,\langle\mathcal{R}_{m+1,m}(\hat{s},\hat{s}')\rangle\mathbf{I}_m^d(z_m,\hat{s}')+\int d\Omega'\,\langle\mathcal{T}_{m,m+1}(\hat{s},\hat{s}')\rangle\mathbf{I}_{m+1}^u(z_m,\hat{s}'). \tag{3}$$

The boundary conditions on the $(m-1)$–th interface are given as

$$\mathbf{I}_m^d(z_{m-1},\hat{s})=\int d\Omega'\,\langle\mathcal{R}_{m-1,m}(\hat{s},\hat{s}')\rangle\mathbf{I}_m^u(z_{m-1},\hat{s}')+\int d\Omega'\,\langle\mathcal{T}_{m,m-1}(\hat{s},\hat{s}')\rangle\mathbf{I}_{m-1}^d(z_{m-1},\hat{s}'), \tag{4}$$

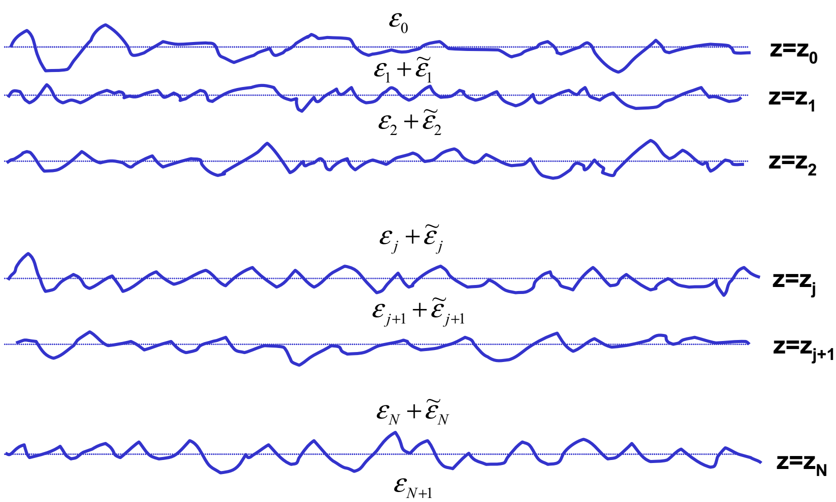

Fig. 1. Geometry of the problem.