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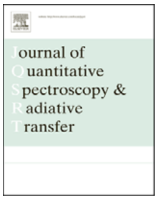

Matrix formulations of radiative transfer including the polarization effect in a coupled atmosphere–ocean system

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ABSTRACT

A vector radiative transfer model has been developed for a coupled atmosphere–ocean system. The radiative transfer scheme is based on the discrete ordinate and matrix operator methods. The reflection/transmission matrices and source vectors are obtained for each atmospheric or oceanic layer through the discrete ordinate solution. The vertically inhomogeneous system is constructed using the matrix operator method, which combines the radiative interaction between the layers. This radiative transfer scheme is flexible for a vertically inhomogeneous system including the oceanic layers as well as the ocean surface. Compared with the benchmark results, the computational error attributable to the radiative transfer scheme has been less than 0.1% in the case of eight discrete ordinate directions. Furthermore, increasing the number of discrete ordinate directions has produced computations with higher accuracy. Based on our radiative transfer scheme, simulations of sun glint radiation have been presented for wavelengths of 670 nm and 1.6 μm. Results of simulations have shown reasonable characteristics of the sun glint radiation such as the strongly peaked, but slightly smoothed radiation by the rough ocean surface and depolarization through multiple scattering by the aerosol-loaded atmosphere. The radiative transfer scheme of this paper has been implemented to the numerical model named *Pstar* as one of the *OpenCLASTR/STAR* radiative transfer code systems, which are widely applied to many radiative transfer problems, including the polarization effect.

1. Introduction

The top of the Earth's atmosphere is irradiated by solar radiation, emitting thermal infrared radiation to space. That radiation interacts with molecules and particles that are present in the terrestrial atmosphere, and reacts also with the ground surface. Especially, the solar radiation is unpolarized at the top of the atmosphere, but the state of polarization and the radiant intensity are changed because of the radiative transfer process. The process, including the polarization effect, follows the radiative transfer equation [1].

The discrete ordinate approach is one technique to solve the radiative transfer equation and obtains the radiation field for given boundary conditions. By discretizing the angular coordinates of the phase function and radiance with respect to the zenith angle, the radiative transfer equation is reduced to a linear differential matrix equation. The ways of solving the matrix equation have been studied for the case of multilayered system with underlying ground surface [2–4]. Based on the solutions, the atmospheric radiative transfer (RT) models have been developed for several purposes such as the radiation budget and remote sensing studies. The scalar RT code

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Nomenclature

α	scattering angle defined in Eq. (36)
α_l, β_l, γ_l, δ_l, ε_l, ζ_l	Greek constants, which are expansion coefficients of $\tilde{\mathbf{P}}$
$\boldsymbol{\alpha}$, $\boldsymbol{\beta}$	integral constants of homogeneous solution
a_1, a_2, a_3, a_4, b_1, b_2	elements of scattering phase matrix
$A^l_{m,n}$	coefficient of the generalized spherical function
$\mathbf{A}^{(m)}$	matrix defined in Eq. (16)
$\mathbf{A}^{\pm}$, $\mathbf{B}^{\pm}$	base function for the scaled Stokes vector defined in Eqs. (82a) and (82b)
β, β_t	incidence and scattering angles with respect to the surface normal vector
b_n, $\mathbf{b}_n$, $\mathbf{B}_n$	expansion coefficients for Planck's function defined in Eqs. (74), (77b), and (77a)
B	Planck function
$\mathbf{B}$	thermal emission vector
$\mathbf{B}_l$	coefficient matrix for $\mathbf{A}^{(m)}$, which contains the Greek constants
$\boldsymbol{\gamma}$	coefficients of particular solution $\boldsymbol{\xi}_S$ defined in Eq. (73)
$\mathbf{c}_n$	expansion coefficients for Planck function defined in Eq. (78)
$\mathbf{C}$	base function defined in Eq. (69a)
$\mathbf{C}_0$	inner product of $\mathbf{U}$ and $\mathbf{V}$
$\mathbf{C}_1$, $\mathbf{C}_2$	Auxiliary matrix defined in Eq. (87)
δ	Dirac delta function
δ_{ij}	Kronecker's delta
$\mathbf{D}$	diagonal matrix for the relation of mirror symmetry
$\hat{\mathbf{D}}$	$\mathbf{D}$ matrix for all discrete ordinate directions
$\mathbf{D}_1$, $\mathbf{D}_2$	auxiliary matrices defined in Eqs. (26a) and (26b)
$\mathbf{D}_n$	coefficient matrix defined in Eq. (84c)
$\boldsymbol{\varepsilon}$	source vector of a single layer
$\mathbf{E}$	exponential functions defined in Eq. (70)
$\mathbf{E}_{0\pm}$	exponential functions defined in Eqs. (45a) and (45b)
$\mathbf{E}_4$	four-order unit matrix
f	delta-M truncation factor
F	auxiliary function defined in Eq. (38)
F_0	extraterrestrial solar flux
$\mathbf{F}$	solar flux vector for several solar zenith angles
$\mathbf{F}_0$	solar flux vector
$\mathbf{g}$	source vector defined in Eq. (63) and used in transfer Eq. (61)
G	shadowing factor
$\mathbf{G}$	coefficient matrix of transfer Eq. (61)
H	Heaviside's step function
$\mathbf{H}$, $\tilde{\mathbf{H}}$	coefficient matrices defined in Eqs. (98a) and (98b) and used in source vector (97)
θ	polar angle
Θ	scattering angle
I, Q, U, V	Stokes parameters
I_p	polarized radiance
$\mathbf{I}$	Stokes vector
$\bar{\mathbf{I}}$	diffused component of the Stokes vector
$\mathbf{I}_1^{(m)}$, $\mathbf{I}_2^{(m)}$	azimuthally separated Stokes vector defined in Eqs. (28a) and (28b)
$\mathbf{I}_1$	single scattering component of the Stokes vector
$\tilde{\mathbf{I}}_1$	accurate single scattering solution using the complete phase matrix
$\mathbf{I}_{\mathrm{TMS}}$	Stokes vector, whose single scattering component are corrected
$\mathbf{J}$	total source vector
$\boldsymbol{\Lambda}^2$	Eigenvalue of matrix $\mathbf{G}$
l, m, n	integral numbers
$\mathbf{L}$	rotation matrix
μ	cosine of polar angle
$\tilde{\mu}$	cosine of the polar angle in the ocean
μ_n	cosine of zenith angle for surface normal vector
μ_i, w_i	double-Gaussian quadrature points and weights
m_a, m_o	real parts of the refractive index of the air and the seawater
$\tilde{m}$	relative refractive index
M	number of the solar zenith angles
$\mathbf{M}$	matrix of quadrature points defined in Eq. (43)
ν	auxiliary parameter defined in Eq. (39)
N	number of quadrature points (discrete ordinate directions)
$\boldsymbol{\xi}$	Stokes vector in eigenspace of $\mathbf{G}$
$\boldsymbol{\xi}_{\mathrm{H}}$	homogeneous solution of transfer Eq. (68)
$\boldsymbol{\xi}_{\mathrm{S}}$, $\boldsymbol{\xi}_{\mathrm{B}}$	particular solution of transfer Eq. (68)
p	probability density function of the wave slope
P_l^m	associated Legendre function
$P^l_{m,n}$	generalized spherical function
$\mathbf{P}$	phase matrix
$\tilde{\mathbf{P}}$	scattering phase matrix
$\mathbf{P}_c^{(m)}$, $\mathbf{P}_s^{(m)}$	cosine and sine series of phase matrix
$\mathbf{P}_l^m$	expansion matrix for $\mathbf{A}^{(m)}$
$\mathbf{P}^*$	truncated phase matrix
R_l^m, T_l^m	auxiliary functions defined in Eqs. (21a) and (21b)
$\tilde{\mathbf{r}}$	Fresnel reflection matrix for the flat ocean surface
$\mathbf{R}$	reflection matrix of a single layer
$\mathbf{R}_{\mathrm{S}}$	reflection matrix of the ocean surface
$\tilde{\mathbf{R}}$	reflection matrix of the rough ocean surface
σ^2	mean square slope of wave facets
$\boldsymbol{\sigma}_{\mathrm{S}}$, $\boldsymbol{\sigma}_{\mathrm{B}}$	source vector defined in Eqs. (59) and (60) and used in transfer Eq. (57)
$\mathbf{S}$	base function defined in Eq. (69b)
$\mathbf{S}_{\mathrm{S}}$	source vector of single scattering
$\mathbf{S}_{\mathrm{B}}$	source vector of thermal emission
$\mathbf{S}_k^{(m)}$	azimuthally separated source vector defined in Eq. (30)
$\mathbf{S}^{(m)}_{\mathrm{S}+,k}$, $\mathbf{S}^{(m)}_{\mathrm{S}-,k}$, $\mathbf{S}^{(m)}_{\mathrm{B},k}$	source terms defined in Eqs. (31a)–(31e)
τ	optical depth
τ_a	optical depth from the top of the atmosphere to the ocean surface
τ_c	optical thickness of a single layer
$\tilde{\mathbf{t}}$	Fresnel transmission matrix for the flat ocean surface
$\mathbf{T}$	transmission matrix of a single layer
$\mathbf{T}_{\mathrm{S}}$	transmission matrix of the ocean surface