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An "exact" geometric-optics approach for computing the optical properties of large absorbing particles

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ABSTRACT

Based on the principles of geometric optics, the ray-tracing technique has been extensively used to compute the single-scattering properties of particles whose sizes are much larger than the wavelength of the incident wave. However, the inhomogeneity characteristics of internal waves within an absorbing particle, which stem from a complex index of refraction, have not been fully taken into consideration in the geometric ray-tracing approaches reported in the literature for computing the scattering properties of absorbing particles. In this paper, we first demonstrate that electromagnetic fields associated with an absorbing particle can be decomposed into the TE and TM modes. Subsequently, on the basis of Maxwell's equations and electromagnetic boundary conditions for the TE-mode electric field and the TM-mode magnetic field, we derive generalized Fresnel reflection and refraction coefficients, which differ from conventional formulae and do not involve complex angles. Additionally, a recurrence formulism is developed for the computation of the scattering phase matrix of an absorbing particle within the framework of the conventional geometric ray-tracing method. We further present pertinent numerical examples for the phase function and the degree of linear polarization in conjunction with light scattering by individual absorbing spheres, and discuss the deviation of the geometric optics solutions from the exact Lorenz–Mie results with respect to size parameter and complex refractive index.

1. Introduction

The single-scattering properties of nonspherical particles are fundamental to various applications in remote sensing research and climate radiative forcing analysis involving aerosols and clouds containing ice crystals [1,2]. In the last three decades, the conventional geometric ray-tracing technique [3–9] and its improved forms [10,11] have been extensively used to compute the optical properties of nonspherical dielectric particles much larger than the wavelength of the incident wave. However, in the geometric ray-tracing methods reported in the literature for computing the single-scattering properties of absorbing dielectric particles, a unique physical property of localized waves within an absorbing particle, referred to as wave inhomogeneity, has not been fully considered.

The inhomogeneity of an electromagnetic wave is related to the wave characteristics such that the planes of constant phase are not parallel to those of constant amplitude [12–16]. This wave feature leads to complex angles in the

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conventional Snell law that defines the incident, reflection and refraction directions. However, a complex angle involving an imaginary number does not have a straightforward geometric meaning within the context of ray-tracing calculations in the real-number domain for the propagation directions of geometric optics rays. Moreover, an inhomogeneous electromagnetic wave does not satisfy a simple transverse-wave condition that requires the corresponding electric and magnetic field vectors to be perpendicular to the propagation direction of constant phase or amplitude. For a non-absorbing particle, electromagnetic waves within the particle are transverse waves for which the three directions along the electric field vector, magnetic field vector and wave propagation are orthogonal. However, in the case of an absorbing particle, an internal electromagnetic wave can be decomposed into a transverse electric (TE) component and a transverse magnetic (TM) component. The electric vector is perpendicular to the direction of wave propagation in the TE mode, whereas the magnetic vector is perpendicular to the direction of wave propagation in the TM mode.

In this study, we derive the generalized Fresnel formulas for the TE-mode and TM-mode field components for various orders of reflection–refraction events associated with an absorbing particle. In the generalized Fresnel formulas for the calculations of the electric and magnetic field amplitudes associated with the reflected and transmitted rays, we follow the effective refractive index concept [17] to circumvent the difficulty that stems from the complex angles in the conventional Snell law for absorbing particles. We also formulate a geometric ray-tracing approach in terms of the generalized Fresnel formulas coupled with various transformation matrices to fully account for the wave inhomogeneity effects on the reflection and refraction of a localized wave characterized as a geometric ray. Furthermore, we present some pertinent numerical examples and compare the geometric optics solutions to the Lorenz–Mie results for the phase function and the degree of linear polarization in conjunction with light scattering by individual absorbing spheres for size parameters in the geometric optics regime.

2. Generalized Fresnel formulas for inhomogeneous waves

In this section, we derive the Fresnel formulas for the electric and magnetic fields in the TE and TM modes, respectively, on the basis of the general electromagnetic boundary conditions for two adjacent media with a plane interface and the two Maxwell curl equations [18] given by

$$\nabla \times \vec{E}(\vec{r},t) = -\frac{\mu}{c}\frac{\partial \vec{H}(\vec{r},t)}{\partial t}, \tag{1}$$

$$\nabla \times \vec{H}(\vec{r},t) = \frac{\varepsilon}{c}\frac{\partial \vec{E}(\vec{r},t)}{\partial t}, \tag{2}$$

where μ and ε are permeability and permittivity, respectively. For our purpose, we should consider a non-ferromagnetic particle such that $\mu = 1$. In this case, the permittivity is then given by

$$\varepsilon = m^2, \tag{3}$$

where m is the index of refraction which is 1 outside the particle and a complex quantity within the particle. We further assume that the time-dependence of the waves is harmonic and can be expressed in the form of $e^{-ik_o ct}$, where $k_o = 2\pi/\lambda$ and λ is the wavelength in a vacuum. For formulation simplicity, in the following discussions we will only consider the spatial components of the electromagnetic fields. Thus, for the spatial domain outside the particle, we have

$$\nabla \times \vec{E}(\vec{r}) = ik_o \vec{H}(\vec{r}), \tag{4}$$

$$\nabla \times \vec{H}(\vec{r}) = -ik_o \vec{E}(\vec{r}). \tag{5}$$

But for the spatial domain inside the particle, the two curl equations are given by

$$\nabla \times \vec{E}(\vec{r}) = ik_o \vec{H}(\vec{r}), \tag{6}$$

$$\nabla \times \vec{H}(\vec{r}) = -ik_o m^2 \vec{E}(\vec{r}). \tag{7}$$

Fig. 1 defines the geometric configuration for the first-order reflection–refraction event when the transmission of an incident ray is from air into the particle. The unit vectors $\hat{e}_{i,1}$, $\hat{e}_{r,1}$ and $\hat{e}_{t,1}$ represent the directions of the incident, reflected and refracted rays, respectively; $\hat{\beta}_1$ is a unit vector with the direction pointing onto the paper; $\hat{e}_{f,1}$ is a unit vector on the incident plane, a plane containing the incident, reflected and refracted rays and parallel to the particle surface; $\hat{n}_1$ is a local unit vector normal to the particle surface; the unit vectors $\hat{\alpha}_{i,1}$, $\hat{\alpha}_{r,1}$ and $\hat{\alpha}_{t,1}$ are defined by the following relations:

$$\hat{\alpha}_{i,1} = \hat{e}_{i,1} \times \hat{\beta}_1, \tag{8}$$

$$\hat{\alpha}_{r,1} = \hat{e}_{r,1} \times \hat{\beta}_1, \tag{9}$$