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General derivation of the total electromagnetic cross sections for an arbitrary particle

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ABSTRACT

This work concerns a common problem in electromagnetic scattering; calculation of the total scattering, extinction, and absorption cross sections for an arbitrary particle. Typical expressions for the cross sections are obtained in terms of the vector spherical wave function expansions for the incident and scattered waves. The unique aspect of this work is that the derivation is carried out specifically without use of the far-field zone approximation. The resulting expressions, valid at any distance, exactly match those obtained from the far-field approximation. This demonstrates that the cross sections are independent of the distance from the particle at which they are calculated as one would expect from energy conservation. Numerical simulations of the near and far-field zone energy flows due to a spherical particle are presented to illustrate several implications of this result.

1. Introduction

The total extinction C^{ext}, scattering C^{sca}, and absorption C^{abs} cross sections for a particle residing in a *nonabsorbent* medium are nearly always calculated in the particle's far-field zone. This is done partly because the mathematical form of the scattered fields are substantially simpler in the far-field zone, and because most cross section measurements are performed far enough from the particle that the far-field approximation can be quite good. The essential physical significance of the total cross sections is that they collectively account for the redistribution and conservation of energy in the scattering process. Consequently, since the external medium is nonabsorbent, one would expect that expressions for these cross sections should be independent of the distance from the particle at which they are calculated. The intent of this work is to explicitly demonstrate how exact expressions for the total cross sections can be found that are independent of distance from the particle. This is done by explicitly evaluating integral expressions for the cross sections in a particle's near-field zone. The exact equivalence of the resulting expressions to those that are derived in the far-field zone proves the distance independence. Numerical simulations of spherical particles are presented that verify the equivalence of the cross sections when calculated in the near and far-field zones and help illustrate the physical significance of this result.

It should be noted that this work is not the first to calculate a particle's cross sections in the near-field. For example, work by Grandy and Bohren and Huffman mentions that C^{sca} and C^{abs} can be calculated for a spherical particle using a surface, like S_{en} described below, that is of arbitrary size but does not show the details of the calculation [1,2]. Videen et al.

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and Fu et al. find near-field expressions for the cross sections for a particle embedded in an *absorbing* medium [3,4]. Further relevant work related to particles in an absorbing medium can be found in [5] and references therein. To the authors' knowledge, the following calculations are the first to consider an *arbitrary* particle and formulate and evaluate exact expressions for the total cross sections valid at *any* distance.

2. Derivation of the cross sections

Consider a fixed arbitrarily shaped particle residing in vacuum and illuminated by a linearly polarized incident plane wave. The particle's complex-valued refractive index m is also arbitrary. The particle is centered at the coordinate origin $\mathcal{O}$ and its surface, internal volume, and external region are denoted by S, V^{int} and V^{ext}, respectively, see Fig. 1. Surrounding the particle is its smallest circumscribing sphere S_{sc} of radius R_{sc}. The purpose of this sphere is to define the minimum distance from the origin beyond which one is guaranteed that the vector spherical wave function (VSWF) expansions of Eqs. (5), (6), (8), and (9) below will converge, see [6]. Also enclosing the particle, and S_{sc}, is another spherical surface S_{en} of radius R_{en}. This sphere will be used as the closed surface over which the fields will be integrated in Eqs. (11) and (12) to yield expressions for the total cross sections.

The wave incident on the particle is a linearly polarized plane wave traveling along the $\hat{\mathbf{n}}^{\text{inc}}$ direction with a wavenumber $k = 2\pi/\lambda$, where λ is the vacuum wavelength. The time dependence of the wave is harmonic and given by $\exp(-\mathrm{i}\omega t)$ where ω is the angular frequency, $\omega = kc$, and c is the speed of light in vacuum. The time dependence will be suppressed in the following for brevity. The electric and magnetic fields of this incident wave are

$$\mathbf{E}^{\text{inc}}(\mathbf{r}) = \mathbf{E}_0^{\text{inc}} \exp(\mathrm{i}kr\hat{\mathbf{n}}^{\text{inc}} \cdot \hat{\mathbf{r}}), \tag{1}$$

$$\mathbf{B}^{\text{inc}}(\mathbf{r}) = \frac{k}{\omega}\hat{\mathbf{n}}^{\text{inc}} \times \mathbf{E}^{\text{inc}}(\mathbf{r}), \tag{2}$$

respectively, where $\mathbf{E}_0^{\text{inc}}$ is a constant vector describing the amplitude and polarization of the wave.

The presence of the particle in the incident wave establishes a new (total) wave that can be *mathematically* regarded as the superposition of the incident and scattered waves. The total electric and magnetic fields outside of the particle are then

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^{\text{inc}}(\mathbf{r}) + \mathbf{E}^{\text{sca}}(\mathbf{r}), \tag{3}$$

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}^{\text{inc}}(\mathbf{r}) + \mathbf{B}^{\text{sca}}(\mathbf{r}). \tag{4}$$

These fields can be expanded into series of VSWFs as

$$\mathbf{E}^{\text{inc}}(\mathbf{r}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[a_{mn}\,\mathrm{Rg}\,\mathbf{M}_{mn}(k\mathbf{r}) + b_{mn}\,\mathrm{Rg}\,\mathbf{N}_{mn}(k\mathbf{r})], \quad \mathbf{r} \in V^{\text{int}} \cup V^{\text{ext}}, \tag{5}$$

$$\mathbf{E}^{\text{sca}}(\mathbf{r}) = \sum_{n=1}^{\infty}\sum_{m=-n}^{n}[p_{mn}\mathbf{M}_{mn}(k\mathbf{r}) + q_{mn}\mathbf{N}_{mn}(k\mathbf{r})], \quad \mathbf{r} \in V^{\text{ext}}. \tag{6}$$

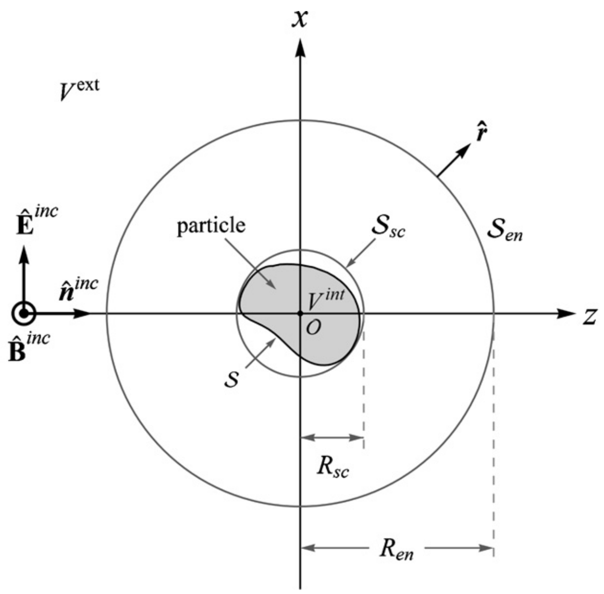

Fig. 1. Scattering arrangement showing an arbitrary particle illuminated by a linearly polarized incident plane wave.