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Approximate calculation of coherent backscattering for semi-infinite discrete random media

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ABSTRACT

We describe an approximate method for the calculation of all characteristics of coherent backscattering for a homogeneous, semi-infinite particulate medium. The method allows one to transform a system of integral equations describing coherent backscattering exactly into a system of linear algebraic equations affording an efficient numerical solution. Comparisons of approximate theoretical results with experimental data as well as with benchmark numerical results for a medium composed of nonabsorbing Rayleigh scatterers have shown that the method can be expected to give a good accuracy.

1. Introduction

Multiple scattering of electromagnetic waves by discrete random media is the focus of interest in various science and engineering disciplines (see, e.g. [1–4] and references therein). At present, the theory of multiple scattering is adequately developed only for the case of sparse media in which scatterers are located in the far-field zones of each other [1–4]. Scattering characteristics of such discrete random media are primarily determined by the so-called ladder and cyclical diagrams [1–3]. The sum of all ladder diagrams characterizes diffuse multiple scattering and reduces to the vector radiative transfer equation. Methods for the solution of this equation in the case of isotropic and homogeneous particulate media in the form of a plane-parallel layer are now well developed, and computer codes for the calculation of the reflection matrix are available on-line (e.g. [5]).

The sum of the cyclical diagrams characterizes coherent multiple scattering which leads to the effect of coherent enhancement of backscattering (see, e.g. [2] and references therein). The calculation of characteristics of this effect constitutes a very complicated problem even in the case of a homogeneous and isotropic particulate layer. This problem is fully solved only for a semi-infinite medium composed of nonabsorbing Rayleigh scatterers provided that the incident radiation propagates normally to the boundary of medium [2,6]. A rigorous equation describing coherent backscattering for a plane-parallel medium consisting of arbitrary randomly oriented and randomly positioned scatterers has been obtained in [7,8]. A qualitative analysis and an approximate numerical solution of this equation based on retaining only several initial orders of scattering show a strong dependence of coherent backscattering on properties of the particles forming the scattering medium [8–10]. This factor is important for the interpretation of remote-sensing data obtained for various

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objects of interest. The exact numerical solution of the equation for the coherent part of the scattered radiation represents an extremely complicated problem. Therefore, numerical modeling of coherent backscattering is sometimes based on Monte-Carlo methods [11]. Unfortunately, this requires considerable expenses of computer time and becomes impracticable in the case of a medium composed of nonspherical particles.

In this paper we propose a simple approximate method for the numerical solution of the equation for the coherent part of the reflected radiation in the case of a semi-infinite medium illuminated by external radiation propagating perpendicularly to the boundary of the medium. Comparisons of approximate theoretical results with laboratory data as well as with benchmark numerical results obtained for a medium composed of nonabsorbing Rayleigh scatterers will demonstrate that the new approach is sufficiently accurate for many practical applications.

2. Basic definitions and equations

Let a homogeneous and isotropic semi-infinite particulate medium be illuminated by a plane electromagnetic wave propagating perpendicularly to the boundary of the medium. In this case the coherent part of the reflected radiation is described by the following matrix (see, in [8, Eqs. (46)–(49)]):

$$S^{(C)}_{pn\mu\nu} = \frac{2\pi\upsilon^2}{k_0^4} \sum_{qq_1LM} (-1)^L \zeta^{*(q_1\mu)(qp)}_{LM} \int_0^\infty \beta^{(z)(qn)(q_1\nu)}_{LM} \exp(-\varepsilon z)\,\mathrm{d}z, \tag{1}$$

where the matrix $S^{(C)}_{pn\mu\nu}$ is defined per unit area of the boundary of the medium, $p, n, \mu, \nu, q, q_1 = \pm 1$, $k_0 = 2\pi/\lambda$, is the wave number, λ is the wavelength of the incident radiation, υ is the particle number density, the asterisk denotes complex conjugation,

$$\varepsilon = \mathrm{Im}(\eta)\left(1 - \frac{1}{\cos\vartheta}\right) + \mathrm{i}(1+\cos\vartheta)\left(\frac{\mathrm{Re}(\eta)-1}{\cos\vartheta} + 1\right), \tag{2}$$

η is the complex effective refractive index of the medium, and ϑ is the scattering angle (Fig. 1). Eq. (1) assumes the use of the circular-polarization representation of the Stokes column vector.

The coefficients $\beta^{(z)(qn)(q_1\nu)}_{LM}$ are determined from the following system of equations:

$$\beta^{(z)(pn)(\mu\nu)}_{LM} = \exp(-\varepsilon^* z) B^{(z)(pn)(\mu\nu)}_{LM} + \frac{2\pi\upsilon}{k_0^3} \sum_{qq_1lm} i^{M-m} \chi^{(pq)(\mu q_1)}_{l} \int \beta^{(y)(qn)(q_1\nu)}_{lm} \times \exp(-\tau\rho) d^L_{MN_0}(\omega) d^l_{mN_0}(\omega) J_{m-M}(\rho\sin\vartheta\sin\omega)\sin\omega\,\mathrm{d}\omega\,\mathrm{d}\rho. \tag{3}$$

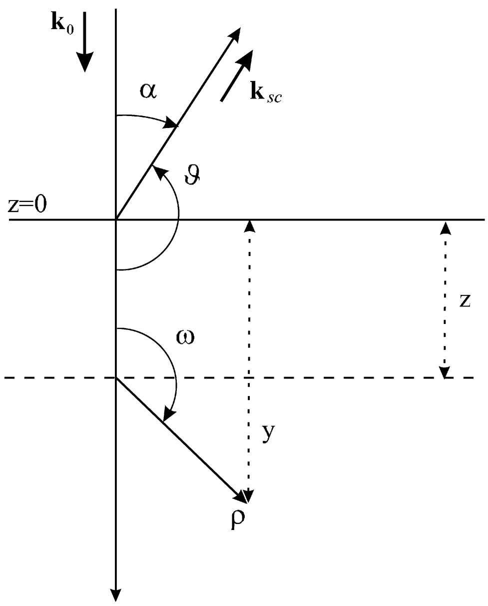

Fig. 1. Geometry of scattering by a semi-infinite medium. The incident light propagates normally to the boundary of the medium ($z = 0$). The directions of incidence and scattering are indicated by the vectors $\mathbf{k}_0$ and $\mathbf{k}_{sc}$, respectively.