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Mueller-matrix model of an inhomogeneous, linear, birefringent medium: Single scattering case

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Abstract

Using the anisotropic phase-screen method, we derive the generalized Mueller matrix of an inhomogeneous, linear, birefringent medium in the single-scattering case. We show that the Mueller matrix include eight nonzero elements with relations among elements given by $m_{11} = m_{22}$, $m_{12} = m_{21}$, $m_{33} = m_{44}$, and $m_{34} = m_{43}$. Using the derived Mueller-matrix model, we simulate polarization optics for calcite, quartz, and sodium nitrate crystals. At $\lambda = 0.63\,\mu\text{m}$ and with small inhomogeneities, the matrix elements are a function of the standard deviation of the crystal thickness, while m_{12} shows pronounced sensitivity to the difference of refractive indices. At an observation angle of 0°, inhomogeneous birefringent media behaves as if they are partial polarizers. Matrix elements most sensitive to the inhomogeneity in the UV to near IR wavelengths have been determined.

Keywords: Phase screen; Coherence matrix; Jones matrix; Mueller matrix; Refractive indices

1. Introduction

Despite the importance of polarized light scattering in the optics of the ocean, atmosphere, and plants (see, for example, [1–8]), the description of light scattering from such media are incomplete. Previous works have demonstrated that scattered radiation is useful for optical diagnosis. The interaction of radiation with an inhomogeneous medium not only randomizes the direction of propagation but also alters the polarization state of the radiation. This means that a complete description of the interaction of polarized radiation with an inhomogeneous medium requires the use of matrix methods.

The objectives of this paper are to derive a Mueller-matrix model of an inhomogeneous, linear, birefringent medium for the single scattering case, and to perform computer simulations using the model. The paper has been organized as follows. Section 2 is devoted to the theoretical derivation of the Mueller-matrix model. Section 3 gives model results for zero observation angle and matrix elements versus wavelength and inhomogeneity. In the end, the matrix elements most sensitive to inhomogeneity are given.

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Nomenclature

$\mathbf{E}$	Jones vector
$\mathbf{T}$	Jones matrix
$\mathbf{G}$	coherence matrix
$\mathbf{\Phi}$	tensor that describes transformation of incident coherence matrix into out coherence matrix
$n_{1,2}$	refraction indices for radiation linearly polarized on x-(horizontal) and y-(vertical) axis, respectively
a	beam radius of incident Gauss field distribution
$\bar{h}$	mean thickness
k	wavenumber

Greek letters

σ_h	mean-square thickness deviation, representing inhomogeneity
σ_{ij}	mean-square phase deviation
$\boldsymbol{\rho}$	radius-vector in the plane orthogonal to the z-axis
$\Sigma^{(i)}$	Pauli matrices
λ	wavelength

Subscript/superscript

eigen	refers to eigenvector coordinate set
in	refers to incident radiation
out	refers to radiation at distant point
scr	refers to radiation behind the phase screen

2. Theory

The geometry of the problem is shown in Fig. 1. Monochromatic radiation is incident perpendicular to the thin parallel-sided slab of birefringent medium located at $z = 0$. The slab has infinite X–Y extent. The output side (where radiation exits) of the slab has random roughness. Hence, the slab performs spatial alterations of phase shift between the electric vector components of radiation resulting in random changes of the polarization ellipse. In other words, the slab acts as an anisotropic phase screen. We calculate polarization states of electromagnetic radiation at distance z (far zone) from the slab and derive the generalized Mueller matrix for inhomogeneous linear birefringent medium in the single scattering case.

If the polarization of electromagnetic radiation is described by the two-component Jones vector [9–12]

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix}, \tag{1}$$

where, E_x and E_y are two orthogonal components of electric vector of radiation in the plane of the screen, then radiation behind the screen is described by

$$\mathbf{E}^{\mathrm{scr}}(\boldsymbol{\rho}) = \mathbf{T}(\boldsymbol{\rho})\mathbf{E}^{\mathrm{in}}(\boldsymbol{\rho}), \tag{2}$$

where the local Jones matrix, $\mathbf{T}(\boldsymbol{\rho})$, is defined by the local width $h(\boldsymbol{\rho})$ and the anisotropic properties of the material in the phase screen. Note that in Eq. (2), Jones vector $\mathbf{E}^{\mathrm{scr}}(\boldsymbol{\rho})$ is given in the local coordinate system, which can generally be different from the laboratory coordinate system shown in Fig. 1 [11]. Input and output radiations in the far zone are examined in the laboratory coordinate system. Anisotropy properties of the