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Abel inversion using Legendre polynomials approximations

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Abstract

An improved Abel inversion method based on Legendre polynomials approximations is presented for reconstructing the original radial distribution of plasma emission coefficients from projected intensities. The method uses the technique of overlapping two near segments for obtaining an excellent approximation of the intensity distribution. The approximated function of the intensity profile is a combination of various shifted Legendre polynomials which in the Abel inverse equation can be integrated exactly to deduce the emission coefficient. It is shown, using simulated intensity data with and without noise inverted for a comparison with those obtained by other methods, that the method is more accurate and has a better property of noise resistance. It is well suited for applying to experimental intensities distorted by noise.

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Keywords: Abel inversion; Function approximation; Legendre polynomial; Noise resistance

1. Introduction

In many areas of physics and engineering, such as plasma diagnostics and flame studies, a commonly encountered problem is the deduction of radial distributions of some important quantity from measurements of the line-of-sight projected values. Typically, for a cylindrically symmetric, optically thin plasma source [1], the relation between the radial distribution of the emission coefficient and the intensity measured from outside of the source is described by the Abel transform (see Fig. 1). And the reconstruction of the emission coefficient from its projection is known as Abel inversion.

Abel inverse integral equation cannot be used directly because the measured intensity data are sets of discrete values. Furthermore, Abel inversion is an ill-posed problem and the derivative of the projection greatly amplifies uncertainties, which make the method is considerably difficult for applying to experimental data in the presence of noise. For these reasons, a large number of numerical methods have been developed over the years. For an ideal numerical method the basic quality should be that it is accurate, stable and computationally efficient. Several studies [2–4] use spline interpolation technique to represent the emission or intensity profile, then perform the inversion based on the Abel transform or the inverse Abel transform.

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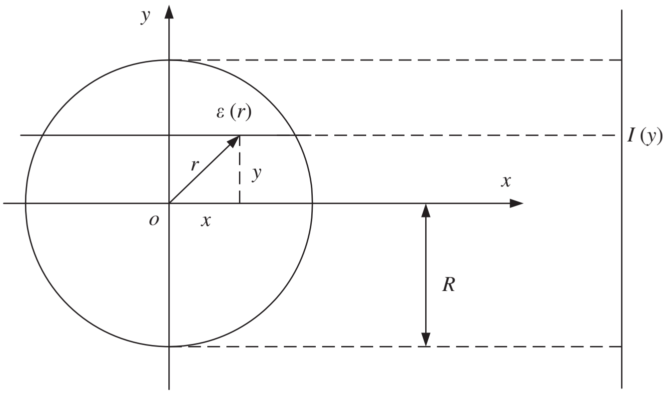

Fig. 1. Schematic diagram showing the coordinate system and geometry of a cylindrically symmetric radiation source with respect to the z-axis, which is normal to the paper.

Another technique used by many researchers [5–7] is the Fourier Hankel transform, with which the emission coefficient of the plasma can be obtained by first taking the Fourier transform of the intensity data, then following with the inverse Hankel transform, which is known as the Fourier–Hankel method. This method is said to be more accurate than those using spline interpolation technique. However, in all the methods the influence of noise on the inverted result is not considered. When noise is presented in the intensity data, the inverted emission profile is undesirable: noise is greatly magnified and oscillations are introduced. Some other approaches based on iterative schemes [8,9] are reported to be more stable on the existence of noise. But the iterative method is time consuming, and when the noise is to be a magnitude larger, it seems to be a problem to use. Algorithms using Gaussian basis expansion techniques [10–12] have also been developed. In fact, they either are difficult to determine the expansion coefficients or accumulate noise towards the center when experimental data have a low signal-to-noise ratio. Polynomial least-squares fitting methods [13–15] are efficient to suppress noise, but in most cases the fitted profile cannot reflect the real distribution of the intensity data.

Taking a smoothing procedure before Abel inversion can significantly reduce the influence of noise, but in fact smoothing is quite a difficult work as there is no mathematical standard can be used to control the smoothing degree. According to the property of Abel inversion, little error in the intensity data will be greatly magnified and accumulated towards the center. Small residual noise after smoothing will produce fluctuations in the inverted data, and over-smoothing will undoubtedly distort the original distribution. Thus noise must be completely removed from the experimental data before taking the inversion with the method which is sensitive to noise. However, noise smoothing methods such as Fourier frequency smoothing technique [6,16], iterative convolution method [17,18] and Savitzky–Golay filters [19], all of them are difficult to be used for a complete removing of noise without corrupting the experimental data. Therefore, it would be highly desirable to develop a inversion method that is accurate and not sensitive to noise, in such a way that there is no need for completely removing the noise.

Function approximation is an effective method to represent profiles. It has been successfully used in many areas, such as the use of wavelets expansion to solve radiative transfer problems [20,21] and the inverse of Abel transform based on using Gaussian functions basis set [10]. There are also methods using Legendre wavelets to compute the Abel inversion [22,23]. The algorithm presented in Ref. [23] based on Legendre wavelets expansion (LWE) is less sensitive to noise than other Abel inversion methods, but for experimental data seriously distorted by noise large error appears in regions near both ends of each approximated segment.

In this paper an improved Abel inversion method using Legendre polynomials approximations (LPA) that combines the advantages of polynomial least-squares fitting and function approximation has been developed for reconstruction of plasma emission coefficients from experimentally measured intensity data. Polynomial least-squares fitting and segments overlapping techniques make the method less sensitive to noise, meanwhile the use of cubic interpolation technique for calculating the approximation coefficients improves the accuracy of the method. Thus this method still yields rather good results even applied to intensity data which have a very low signal-to-noise ratio.