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Analytically derived conversion of spectral band radiance to brightness temperature

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Abstract

Simple analytic expressions for brightness temperature have been derived in terms of band response function spectral moments. Accuracy measures are also derived. Application of these formulas to GOES-12 Sounder thermal infrared bands produces brightness temperature residuals between −5.0 and 2.5 mK for a 150–400 K temperature range. The magnitude of residuals for the five ASTER Radiometer thermal infrared bands over the same temperature range is less than 0.22 mK.

Keywords: Radiation transport; Brightness temperature; Thermal infrared; Spectral band; Blackbody

1. Introduction

Spectral band brightness temperature (BT) is the temperature a blackbody needs to have to emit a specified radiance for a given sensor band or instrument slit function. Generally, the convolution of the Planck function with a band's normalized spectral response (NSR) function cannot be analytically inverted to determine BT. Three standard approaches are used to estimate BT. In the simplest algorithm, a *center* wavenumber or wavelength is defined, and the Planck function is inverted at that spectral point. As is demonstrated below, the first spectral moment of the NSR function is an *optimal* choice for the center spectral point. Iterative approaches for determining BT enable arbitrarily fine accuracy requirements to be met, but these approaches are computationally inefficient. Empirical fits of BT versus radiance, each dependent on spectral channel, are typically used for processing of multispectral sensor data [1,2]. These parameterizations provide high accuracy with efficient computation. Setup includes specifying an appropriate parametric form and performing a parameter optimization for each spectral channel over a chosen temperature range.

For radiative transfer models not specifically designed around a given sensor, such as the MODTRAN™ atmospheric band model [3], it is convenient to have an accurate expression for converting spectral band average radiance to BT that does not require a parameter optimization for each distinct spectral response function. MODTRAN™ already includes an option to read in arbitrary tabulated spectral response functions and generate spectrally convolved transmittances and radiances. MODTRAN™ output also

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includes line-of-sight spectral radiances for any of a number of slit function types with a user-specified spectral resolution (full-width at half-maximum, FWHM). In this paper, BT is expressed as a function of analytic rather than fit parameters. The analytic parameters are NSR function spectral moment and moment difference integrals, which are computed as spectral response functions read in for transmittance filter calculations and computed from analytic formulas for the MODTRAN™ slit function spectral radiances. Thus, the BT expressions derived in this paper provide a direct calculation procedure, which avoids the computational cost and difficulties associated with parameter optimizations.

Let $R[f_v]$ be a spectral band average radiance (e.g., $\mathrm{W\,cm^{-2}\,sr^{-1}/cm^{-1}}$) for an NSR function, f_v, dependent on spectral wavenumber, v. The BT then equals

$$\mathrm{BT}_1[f_v] = \frac{c_2 N_1}{\ln\left(1+\frac{1}{\rho}\right) + \langle\delta N_2\rangle\left\{\frac{3}{1+\rho} - \ln\left(1+\frac{1}{\rho}\right)\left[3-\left(\frac{1}{2}+\rho\right)\ln\left(1+\frac{1}{\rho}\right)\right]\right\}}; \quad \rho \equiv \frac{\pi R[f_v]}{c_1 N_1^3}, \tag{1}$$

assuming that the following conditions hold:

$$\langle\delta N_2\rangle^2 \ll 1 \text{ and } \begin{cases} \langle\delta N_4\rangle \ll 1 & \text{if } f_v \text{ is symmetric about } N_1, \\ |\langle\delta N_3\rangle| \ll 1 & \text{otherwise.} \end{cases} \tag{2}$$

In these equations, c_1 and c_2 are the first and second radiation constants. The spectral wavenumber moments, N_m, and moment difference integrals, $\langle\delta N_m\rangle$, are defined by

$$N_m \equiv \int_v v^m f_v\,\mathrm{d}v \text{ with } N_0 = 1; \quad \langle\delta N_m\rangle \equiv \int_v \left(\frac{v}{N_1}-1\right)^m f_v\,\mathrm{d}v$$
$$\Rightarrow \langle\delta N_2\rangle = \frac{N_2}{N_1^2} - 1, \quad \langle\delta N_3\rangle = \frac{N_3}{N_1^3} - \frac{3N_2}{N_1^2} + 2 \quad \text{and} \quad \langle\delta N_4\rangle = \frac{N_4}{N_1^4} - \frac{4N_3}{N_1^3} + \frac{6N_2}{N_1^2} - 3, \tag{3}$$

where $\int_v$ is the integral over the spectral wavenumber domain of the NSR function.

If $R[f_\lambda]$ is a spectral band average radiance (e.g., $\mathrm{W\,cm^{-2}\,sr^{-1}/\mu m}$) for an NSR function, f_λ, dependent on spectral wavelength, λ, then BT equals

$$\mathrm{BT}_1[f_\lambda] = \frac{c_2/\Lambda_1}{\ln\left(1+\frac{1}{\sigma}\right) + \langle\delta\Lambda_2\rangle\left\{\frac{15}{1+\sigma} - \ln\left(1+\frac{1}{\sigma}\right)\left[6-\left(\frac{1}{2}+\sigma\right)\ln\left(1+\frac{1}{\sigma}\right)\right]\right\}}; \quad \sigma \equiv \frac{\pi R[f_\lambda]}{c_1/\Lambda_1^5}, \tag{4}$$

assuming the following conditions hold:

$$\langle\delta\Lambda_2\rangle^2 \ll 1 \text{ and } \begin{cases} \langle\delta\Lambda_4\rangle \ll 1 & \text{if } f_\lambda \text{ is symmetric about } \Lambda_1, \\ |\langle\delta\Lambda_3\rangle| \ll 1 & \text{otherwise.} \end{cases} \tag{5}$$

In these equations, the spectral wavelength moments, Λ_m, and moment difference integrals, $\langle\delta\Lambda_m\rangle$, are defined by

$$\Lambda_m \equiv \int_\lambda \lambda^m f_\lambda\,\mathrm{d}\lambda \quad \text{with} \quad \Lambda_0 = 1, \quad \langle\delta\Lambda_m\rangle \equiv \int_\lambda \left(\frac{\lambda}{\Lambda_1}-1\right)^m f_\lambda\,\mathrm{d}\lambda$$
$$\Rightarrow \langle\delta\Lambda_2\rangle = \frac{\Lambda_2}{\Lambda_1^2} - 1, \ \langle\delta\Lambda_3\rangle = \frac{\Lambda_3}{\Lambda_1^3} - \frac{3\Lambda_2}{\Lambda_1^2} + 2 \quad \text{and} \quad \langle\delta\Lambda_4\rangle = \frac{\Lambda_4}{\Lambda_1^4} - \frac{4\Lambda_3}{\Lambda_1^3} + \frac{6\Lambda_2}{\Lambda_1^2} - 3, \tag{6}$$

where $\int_\lambda$ is the integral over the spectral wavelength domain of the NSR function.

The conditions of Eqs. (2) and (5) essentially state that the width of the spectral band must be small when compared to the band center location, a requirement that is true for the vast majority of infrared and visible spectral bands of operational interest. Both BT expressions (Eqs. (1) and (4)) are computationally very simple. The spectral moment and moment difference integrals are only functions of the NSR function, not specific band radiance values; therefore, these terms can be computed up front. Once the NSR-dependent parameters are defined, evaluation of the BT for a given band radiance only requires calculation of a single logarithm, specifically inversion of the Planck function at the first spectral moment, and a few of additions, subtractions, multiplications and divisions.