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The spatial polarization distribution over the dome of the sky for abnormal irradiance of the atmosphere

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Abstract

The paper deals with the polarized radiative transfer within a slab irradiated by a collimated infinitely wide beam of arbitrary polarized light. The efficiency of the proposed analytical solution lies in the assumption that the complete vectorial radiative transfer solution is the superposition of the most anisotropic and smooth parts, computed separately. The vectorial small-angle modification of the spherical harmonics method is used to evaluate the anisotropic part, and the vectorial discrete ordinates method is used to obtain the smooth one. The azimuthal expansion is used in order to describe the light field spatial distribution for the case of abnormal irradiance and to obtain some known neutral points in the sky especially useful for polarized remote sensing of the atmosphere.

Keywords: Polarized radiative transfer; Complete solution

1. Introduction

It is well known in optics that polarization state of light described by four-element Stokes vector (SV) contains all the information about an object under consideration available for optical methods of remote sensing (RS). Nevertheless today the number of scalar (neglecting polarization) studies is much more than polarimetric ones. This relates with the comparatively small number of polarimetric systems all over the world. And this fact in turn can be explained by two main reasons: design problems in electro-optical polarimetric systems (high accuracy of measurements must be applied to determine the polarization state of light) and mainly by absence of a reliable mathematical model including polarization for interpretation of the experimental results (see Polarization Science and Remote Sensing II, SPIE 5888 (2005), for example—quite many polarimetric systems and simultaneously only a few theoretical investigations). Following the scalar case the polarized radiative transfer (RT) mathematical model must be of high efficiency from the point of view of the numerical solutions's convergence to the exact one. It must allow to compute highly anisotropic scattering of natural formations (clouds, ocean, galaxy dust and others) and must be valid for arbitrary optical thickness τ and the irradiance angle θ_o of a scattering media (the last one allows to describe known directions of neutral

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polarization of atmosphere-scattered light—Arago, Babinet and Brewster points). The model must include multiple scattering and if possible the solution must be expressed in analytical form to make the evaluation of inverse problems a little simpler. This paper deals with a described model applied to a slab irradiated by infinitely wide collimated beam (plain unidirectional (PU) source of radiation with $\hat{\mathbf{l}}_0$ as a direction of the irradiation). The incident light is assumed to be both natural and arbitrary polarized in different calculated examples.

2. The anisotropic part

The approach described here is considered in Ref. [1] both for vectorial but mainly for 3D scalar approximation together with main references. Here we concentrate on the vectorial case more deeply. We will use the following notation: "→" is the four-element column vector; "↔" is the 16-element square Mueller matrix; Λ is the single scattering albedo; θ and φ are zenithal and azimuthal angles respectively; $\mu = \cos\theta$, the unit directionality vector is $\hat{\mathbf{l}}$. The SV and its component we note as $\vec{L} = [I \quad Q \quad U \quad V]^{\mathrm{T}}$, "T" is the transpose operation. In RT one of the main problems is to take mathematical features of the boundary problem for the vectorial radiative transfer equation (VRTE) into account. These features are caused by ray approximation of radiation's propagation description. For the PU-source such mathematical feature is the non-scattered radiation expressed as Dirac δ-singularity. This singularity needs infinite number of elements to be represented in a series and hence cannot be computed analytically. Chandrasekhar separated the light field within the slab into two parts—δ-singularity and scattered light—and computed the diffuse transparent and reflected light field [2]. But for real turbid media, the scattered light field still remains a highly anisotropic function which needs lots of terms of the series to be computed. This leads to the ill-conditionality of the evaluations and besides computation time increases.

We follow with an idea that showed good results for the scalar case [3–6] and represent the desired vectorial radiation field as the superposition of the anisotropic part that includes the δ-singularity and smooth non-small angle part (indexed by "R"—regular part). So we write for the desired spatial distribution of SV (see Fig. 1) the following equation:

$$\vec{L}(\tau, \hat{\mathbf{l}}) = \vec{L}_{\mathrm{R}}(\tau, \hat{\mathbf{l}}) + \vec{L}_{\mathrm{SA}}(\tau, \hat{\mathbf{l}}). \tag{1}$$

We use the definition, the addition theorem and some recurrence formulas from Gelfand [7] for generalized spherical functions (GSF)

$$\overset{\leftrightarrow}{Y}{}^k_m(\mu) = \mathrm{Diag}[P^k_{m,+2}(\mu);\ P^k_{m,+0}(\mu);\ P^k_{m,-0}(\mu);\ P^k_{m,-2}(\mu)],$$

which represent the eigenfunctions for the scattering operator of the VRTE and write down the standard series to express SV and the scattering matrix $\overset{\leftrightarrow}{x}$ as follows (both in circular polarization [8]—CP, usually used in polarized radiative transfer problems):

$$\vec{L}_{\mathrm{CP}}(\tau, \hat{\mathbf{l}}) = \sum_{m=-\infty}^{\infty} \sum_{k=0}^{\infty} \frac{2k+1}{4\pi} \overset{\leftrightarrow}{Y}{}^k_m(\mu) \vec{f}^k_m(\tau) \exp(im\varphi),$$

$$[\overset{\leftrightarrow}{x}(\hat{\mathbf{l}}\hat{\mathbf{l}}')]_{r,s} = \sum_{k=0}^{\infty} (2k+1) x^k_{r,s}(\tau) P^k_{r,s}(\hat{\mathbf{l}}\hat{\mathbf{l}}'). \tag{2}$$

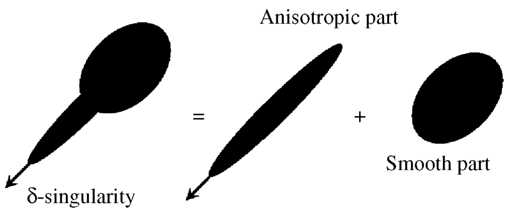

Fig. 1. The superposition of two parts—anisotropic (containing all singularities) and smooth one—schematically.