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A two-step strategy for numerical simulation of radiative transfer with anisotropic scattering and reflection

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Abstract

This article presents a two-step procedure for the computation of radiative heat transfer with anisotropic scattering and reflection. It is based on a concept that the coincident processes of absorption and scattering/reflection can be separated factitiously. All medium elements and wall surfaces are supposed to be pure-absorbing when receiving incident radiation. Afterwards they emit the scattered/reflected radiations. The absorption of both the initial and the secondary radiations can be assessed by the direct exchange area. It is needed to repeat the processes for a few times until the radiations are substantially absorbed. For anisotropic scattering/reflection, a vector summation obtains the directional distribution of emissive power. The method is validated by several benchmark computations in terms of emissive power and heat transfer coefficients. It is shown that the method gives more accurate solution than the isotropic scaling for the heat transfer in anisotropically scattering media.

Keywords: Radiative heat transfer; Participating media; Anisotropic scattering; Exchange area; Zone method; Monte-Carlo method

1. Introduction

Radiation Transfer of participating media has applications in furnace designs, semiconductor processes, optics and environments, to name a few. The conservation law is formulated by a radiative transfer equation (RTE) in a differential-integral form. Problems arise from the modeling or the calculation of the integration term which accounts for scattering effects. Researchers have developed many methods [1,2] for this issue, including exchange-area-based methods (EAM) (e.g., zone method [3], REM^2 [4]), multi-flux methods (e.g., discrete ordinates method [5], finite volume method [6], and YIX method [7]), and stochastic methods (e.g., Monte-Carlo method (MCM) [8]) or their combinations (e.g., discrete transfer method [9]).

The EAM benefit from their simpleness and efficiency. For a medium with constant radiative properties, heat transfer calculation becomes a simple algebraic operation over mesh grids as soon as the exchange area is known. The time taken by equation solving is saved as have to be done by the multi-flux methods. However,

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the EAM's drawbacks are yet multiple, one of which is the difficulty in the operation of anisotropic scattering and non-diffuse wall reflection. Hottel and Sarafim [3] and Noble [10] developed a scheme to calculate total exchange area (TEA) for isotropic scattering. For a system composed of N cells and M surfaces, L ($L=N+M$) equations of L dimensions have to be solved. Following the same concept, the scheme is extended to the simulation of radiation in anisotropically scattering media [11–13], where the method suffers from either too large matrix or too many equations to be solved. This is prohibitive in practice even for the computation using a moderate-scale mesh. One remedy is to use the isotropic-scaling method (e.g., the REM^2 in [14]). Unfortunately, the isotropic scaling of phase function does not work at all time [15,16]. Another disadvantage of EAM is the huge amount of memory for saving exchange area. Both direct exchange area (DEA) and TEA are $L\times L$-dimensional arrays (in view of reciprocity they are $L(L+1)/2$-dimensional) so that the memory is several orders larger than that for solving convection–diffusion equations. In fact, time and memory are just the two limitations for the radiation computation using fine grids.

The MCM provides one more choice for exchange-area calculation. But the simulation of scattering and reflection takes additional time because more rays have to be traced in order to get confident statistic distribution. This increased time will be saved if only DEA is involved.

This paper presents a new DEA-based numerical scheme. It is based on a concept that the coincident absorption and scattering/reflection processes can be fictitiously separated, i.e., (1) at first all the medium elements and wall surfaces are supposed to be pure-absorbing when receiving incidence radiation, and (2) secondly the scattered/reflected radiations are emitted afterwards (say secondary radiations hereafter). Since the absorption of both the initial and secondary radiations can be assessed by DEA, the operation of the scattering/reflection process is mostly simplified. The secondary radiations will be substantially absorbed after a few repeats of the above processes. In the computation only DEA is used and hence the least memory is needed.

The contents are arranged as follows. Section 2 is concerned with the theoretical formulation. Numerical results are reported in Sections 3 and 4, where the model is discussed with regard to some benchmark problems and nonlinear scattering models. Remarks for the applications of the model are given in Section 5.

2. Theoretical formulation

We consider a radiative element of a participating medium, where the radiation intensity I at a position $\vec{r}$ in a direction $\hat{s}$ can be formulated as

$$\frac{\mathrm{d}I(\vec{r},\hat{s})}{\mathrm{d}S}=-(\kappa+\sigma_\mathrm{s})I(\vec{r},\hat{s})+\kappa I_\mathrm{b}(T)+\frac{\sigma_\mathrm{s}}{4\pi}\int_{4\pi}I(\vec{r},\hat{s}')\Phi(\hat{s}\cdot\hat{s}')\mathrm{d}\Omega. \tag{1}$$

Eq. (1) is the RTE, where κ and σ_s are the absorption and scattering coefficients, respectively. $\Phi(\hat{s}\cdot\hat{s}')$ is a phase function, whereas $\hat{s}'$ denotes an incidence direction. Supposing that a wall surface has an emissivity of ε, and a reflectivity of γ, in favor of analysis we define an extinction coefficient α and a scattering/reflection albedo ω, i.e., $\alpha=\kappa+\sigma_\mathrm{s}$, $\omega=\sigma_\mathrm{s}/\alpha$ for medium elements, and $\alpha=\varepsilon+\gamma=1$, $\omega=\gamma/\alpha$ for surfaces. For a system composed of pure-absorbing elements with equivalent absorptivities of α_i ($i,j=1,2,\ldots,L$), the DEA D_{ij} ($i,j=1,2,\ldots,L$) can be calculated by many methods, such as the Zone method, MCM, or a ray-tracing method, etc. The discussion hereafter focuses on the approaches of radiative transfer computation by use of D_{ij}.

2.1. Traditional methods

For a scattering medium bounded by reflection walls, Refs. [3,4,10,11–13] have proposed several ways to conduct the heat transfer simulation using DEA. We give a brief review to those methods. They share the common feature of the procedure proposed initially by Hottel and Sarafim [3], and hence are referred as Hottel's method (HM).

Of an isotropically scattering/reflecting element i, the gross radiosity F_iW_i is a sum of self-emission $F_i(1-\omega_i)E_{\mathrm{b},i}$ and the scattered/reflected radiations $\omega_i\sum_{j=1}^{L}D_{ij}W_j$, i.e.,

$$F_iW_i=F_i(1-\omega_i)E_{\mathrm{b},i}+\omega_i\sum_{j=1}^{L}D_{ij}W_j,\quad i=1,2,\ldots,L, \tag{2}$$