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Chord length distributions in binary stochastic media in two and three dimensions

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Abstract

Simple models for transport through stochastic media usually assume that the chord lengths of materials are distributed exponentially. Theory predicts that, in a medium consisting of disks/spheres that can interpenetrate, chord lengths in the background material (between the disks/spheres) should exactly follow an exponential. In a medium with impenetrable (non-overlapping) disks/spheres, the distribution is only approximately exponential. This paper demonstrates, through direct numerical simulations, that for randomly distributed disks in 2D and spheres in 3D, with distributions of radii, chord lengths in the background material (between the disks/spheres) are accurately described by exponentials over five orders of magnitude when the material is dilute. The chord lengths inside the disks and spheres are not exponentially distributed, but those distributions can be calculated. A scaling relationship between the mean chord lengths in the two materials is presented for an infinite medium. By knowing the mean properties of the disks/spheres in a medium, this relationship allows one to accurately describe the statistical properties of the background material. The stochastic simulations are validated by this infinite medium relationship. When the fraction of space occupied by the disks or spheres becomes large, the distributions are no longer accurately described by an exponential.

Keywords: Stochastic media; Radiation transport; Chord length distributions; Beer–Lambert law; Markovian processes; Poisson processes

1. Introduction

The transport of radiation through random mixtures of two materials has been of interest for many years [1–8]. Applications can vary over a wide range from astrophysical clouds in the interstellar medium to pebble bed nuclear reactors. In modeling the transport, it is often assumed that the distribution of path lengths is Markovian [2,3]. In other words, along any ray through a medium, the material transitions are described by Poisson statistics, so that the probability of a transition from material 1 to material 2 in a short distance ds is

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ds/λ_1. This corresponds to an exponential distribution of material chord lengths with the mean chord length given by λ_1.

In this paper, we consider 2D systems of circular disks randomly positioned in a uniform background material, and 3D systems of spheres randomly positioned in a uniform background material. Refs. [11,12] show theoretically that for overlapping disks and spheres the distribution of chord lengths in the background should be exponential. For non-overlapping disks and spheres, using thermodynamic equilibrium arguments, they show that the distribution should be approximately exponential. Over a limited range, Refs. [9–12] show that chord length distributions are indeed approximately exponential. Here we demonstrate that, for impenetrable disks/spheres with different distributions of disk/sphere sizes in 2D/3D, the distribution of chord lengths in the background material is accurately exponential ($\pm 2\%$) over a dynamic range of more than five orders of magnitude when the material is dilute (less than 10% filling fraction). In this case, the difference between overlapping and non-overlapping disks/spheres is small and the assumption of Poisson statistics in the background material is accurate. Within the disks and spheres, the chord lengths are not exponentially distributed. However, in Appendix A we derive the chord length distributions in 2D for disks, and in Appendix B we derive the chord length distributions in 3D for spheres. Therefore, one can create a complete statistical description of a stochastic mixture of two materials that may be useful for transport calculations.

In atmospheric research, the exponential attenuation of radiation in a homogeneous random medium is known as the Beer–Lambert law. Recent papers [6–8] examine deviations from exponential attenuation that will be discussed where appropriate in the following sections.

The remainder of this paper is organized as follows. Section 2 describes how the stochastic media are constructed. Then Sections 3 and 4 contain the basic 2D and 3D results. An infinite medium scaling relationship is derived in Section 5, followed by a conclusion section. The results presented here are a major extension and improvement of an earlier paper [5].

2. Numerical simulation of stochastic media

The simulation of a stochastic medium in this work is similar to the approach of Donovan and Danon [3] and has been briefly described in Paper I [5]. The major steps in 2D/3D are:

1. Disk/sphere radii are sampled from a specified distribution.
2. Disk/sphere locations are randomly generated in a square/cubical domain.
3. Locations that overlap a boundary or another disk/sphere are rejected and regenerated.
4. The total area/volume of the disks/spheres is a given fraction of the square/cube, p_1. This is the probability that a randomly chosen point will be in material 1.
5. The remaining region is the background material, designated material 2, and will have a probability $p_2 = 1 - p_1$.

When placing disks/spheres with varying radii, it is necessary to place the large ones first. If a large disk/sphere is left to last, there may not be an empty area/volume that is large enough to contain the disk/sphere. This procedure could possibly introduce a subtle correlation effect, but no problems have been observed. The rejection of overlapping disks/spheres will also introduce a slight correlation effect, but this assumption is necessary for modeling some physical systems and is useful in making ray tracing tractable.

Because material 1 is not allowed to overlap the boundaries of the domain, there is a boundary layer in which the statistics are incorrect. Therefore, only the region interior to this layer can be used for generating accurate statistics. The size of this boundary layer is determined by the largest diameter disk/sphere used in the simulation of the medium. For an exponential distribution of radii this diameter would be infinite, so one compromises by using a boundary layer thickness that is 20 times the mean radius for this case. The mean radius used in most simulations (unless otherwise stated) will be $\langle r \rangle = 0.01$. The size of the medium in 2D is chosen such that sampling rays will intersect, on average, 65 disks. In 3D, this would create cubes with too many spheres to handle efficiently. Therefore, in three dimensions slightly smaller domains will be used, as indicated in the following sections.