

Abel inversion using Legendre wavelets expansion

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Abstract

A new method is presented for reconstruction of the radially distributed emissivity from the line-of-sight projected intensity. The method is based on approximating the projected intensity profile by Legendre wavelets. The coefficients of the approximation are computed using the inner product of Legendre wavelets and the intensity profile. The emissivity profile is then obtained by the combination of the functions related to Legendre wavelets. The method is more accurate and noise resistant than other methods when applied to experimental data, and there is no need for a complete noise filtering of the intensity data before applying the inversion.

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1. Introduction

In many applications of physics and engineering, such as astronomy, plasma diagnostics and flame studies, the parameters in the inner part of the study object are almost impossible to be measured directly. In most cases, only the line-of-sight projected intensities can be collected. Therefore, how to find the radial distributions of the studied parameters from the projected data is an extensively concerned problem. For a cylindrically symmetric source, for example an arc plasma with the optically thin assumption, the projected intensity is connected with the emissivity along the line in the measurement direction by a determined function. And the reconstruction of the emissivity distribution from the projected data is known as Abel inversion.

Let $I(y)$ denote the projected intensity as a function of the displacement y from the source center, and $\varepsilon(r)$ is the corresponding emissivity along the radius r of the cylindrically symmetric source. For simplicity, we normalize the radius of the source to 1. Then the Abel transform can be written as

$$I(y) = 2 \int_y^1 \frac{\varepsilon(r)r \, dr}{\sqrt{r^2 - y^2}}. \quad (1)$$

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Since $I(y)$ is assumed to be zero in the area $|y| > 1$, the value of $\varepsilon(r)$ is non-zero only for $r \leq 1$. The expression of $\varepsilon(r)$ is given by the Abel inversion

$$\varepsilon(r) = -\frac{1}{\pi} \int_r^1 \frac{I'(y) dy}{\sqrt{y^2 - r^2}}. \quad (2)$$

Practically, $I(y)$ is measured as a set of discrete values, hence the analytical expression of $\varepsilon(r)$ cannot be solved directly by Eq. (2). Furthermore, the singularity and derivative in Eq. (2) amplify noise and make the inverted results instable. These characteristics involve great difficulties in performing reliable inversions of experimental data. Till now, a variety of numerical inversion methods have been developed to overcome these problems; they mainly use methods of splines [1–3], polynomial least squares fitting [4–9], and Fourier transform techniques [10–13]. Some other techniques such as iterative procedures [14,15] and regularization method [16] are time consuming or mainly suitable for discontinuous Abel inversion and will not be discussed here.

Several numerical methods assume some sort of variation of $\varepsilon(r)$ or $I(y)$ over each subinterval of interval $[0, 1]$, and then use Eq. (1) or (2) to perform the inversion procedure. For the numerical integration of Eq. (1), Maecker [1] assumed $\varepsilon(r)$ to be constant over a subinterval, and Frie [2] used a second-order interpolation formula instead. Similar to these schemes, Bockasten [3] integrated Eq. (2) with an interpolation formula of third order for $I(y)$. Higher order interpolation formula is also possible and gives more accurate results, but the deduced formula of $\varepsilon(r)$ will be very complex and the modified method is more sensitive to noise. In fact, these methods will greatly amplify errors in the input data [6,9], therefore very smooth data are needed before performing inversion.

The polynomial least squares fitting method was first introduced by Freeman and Katz [4]; they fitted all the data with a polynomial of fixed order. But the inverted results are very poor, especially near the edges of the fitted interval [6,9,17]. Using higher order polynomials, the fitted error at each point decreased, but fluctuations were introduced. And for more complicated profiles, the fitted results are unreasonable and thus the method cannot be used. In order to improve the quality of the least squares fitting method, Cremers and Birkebak [6] suggested to break the data into five segments and apply least squares fit of a low order polynomial to each of the segments. This technique is said to be superior than other methods when applied to experimental data. But in practice, it is difficult to determine the optimal number of segments and the order of least squares fitting polynomial for different distribution profiles. And if these parameters are selected improperly, the inverted results will be very unsatisfied. Some other methods [7–9] employed sets of orthogonal polynomials of arbitrary order to deduce the analytical expression of $\varepsilon(r)$. However, they have no difference with least squares fit but are computationally more complex. Methods using Gaussian basis expansion techniques [18,19] have also been developed, but it is time consuming to compute the expansion coefficients in Ref. [18], and making a best choice of the Tikhonov regularization parameter in Ref. [19] is difficult when experimental data have a low signal-to-noise ratio.

Fourier–Hankel method is a lately developed method using fast Fourier transform to perform the Abel inversion. The detailed description of the method and the data processing techniques before inversion can be found in Refs. [12,13]. Some studies [5,20] reported that the method is very accurate even with few data points, but some other researches [19,21] indicated that it will produce artificial structures and is less accurate than other methods for large set of data points. In fact, the Fourier–Hankel method can be deduced from the Fourier expansion method [10,11], of which Kalal and Nugent pointed out that the Fourier expansion method cannot handle data accurately if the derivative does not vanish at the tail of the data profile. And we found that the Fourier–Hankel method, though mathematically avoids the singularity and derivative in Abel inversion equation, is very sensitive to noise and produces fluctuations when used to inverse experimental data.

The theory of wavelet analysis is a relatively recent development in applied mathematics [22]. As a powerful tool, wavelets have been extensively used in signal processing, numerical analysis, and many other areas. The use of wavelets to solve partial differential equations and integro-differential equations can be found in Refs. [23,24], respectively. For the solution of radiative transfer problems, Bayazitoglu and Wang [25] introduced the wavelet expansion method into this area. And later, many researches [26–28] have been done successfully based on this technique, which was demonstrated to be a very effective tool for function approximation.

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