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A sensitivity function-based conjugate gradient method for optical tomography with the frequency-domain equation of radiative transfer

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Abstract

The Sensitivity Function-based Conjugate Gradient Method (SFCGM) is described. This method is used to solve the inverse problems of function estimation, such as the local maps of absorption and scattering coefficients, as applied to optical tomography for biomedical imaging. A highly scattering, absorbing, non-reflecting, non-emitting medium is considered here and simultaneous reconstructions of absorption and scattering coefficients inside the test medium are achieved with the proposed optimization technique, by using the exit intensity measured at boundary surfaces. The forward problem is solved with a discrete-ordinates finite-difference method on the framework of the frequency-domain full equation of radiative transfer. The modulation frequency is set to 600 MHz and the frequency data, obtained with the source modulation, is used as the input data. The inversion results demonstrate that the SFCGM can retrieve simultaneously the spatial distributions of optical properties inside the medium within a reasonable accuracy, by significantly reducing a cross-talk between inter-parameters. It is also observed that the closer-to-detector objects are better retrieved.

Keywords: Optical tomography; Radiative transfer equation; Forward problem; Adjoint problem; Inverse problem; SFCGM; Frequency domain; Cross-talk; Anisotropic scattering

1. Introduction

Monitoring of optical properties, such as scattering and absorption coefficients, inside biological tissues, is very important for diagnostics in biomedical fields. This is because local changes in optical properties are closely related to physiological and pathological changes of tissues and consequently the knowledge of these optical properties makes it possible to diagnose disease or defects in tissues, for example, to locate breast cancer [1–3]. For these reasons, various non-intrusive methods such as magnetic resonance imaging (MRI), computed tomography (CT), X-ray, and so on, have been introduced so far. Among these, the MRI and the CT are most accurate with high spatial resolutions, but they require cost-expensive, complex optical setup; as

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for the use of X-rays, it is known that tissues may be damaged. On the other hand, near-infrared optical tomography (NIROT) has the advantages of requiring relatively cost-effective, simple instrumentation and yielding feasible accuracy. This method also was shown to yield the local maps of optical properties inside a test medium by analyzing light transmission signature at boundary surfaces with an appropriate inversion procedure. Thus the NIROT technique has been emerging as a very promising diagnostic tool in a wide variety of biomedical fields. Moreover, this method produces no physical damage to the test medium because it employs, as a probing source, near-infrared radiation in the range between 600 and 900 nm.

The NIROT is certainly a very powerful imaging modality, but there still exist challenging problems to be solved before this technique can be used on an extensive basis. The main issues of NIROT can be categorized into the followings: an accurate forward model to predict light transport inside the medium, a precise measurement technique to detect the outgoing intensity from boundaries, and a fast, accurate inversion algorithm to retrieve inside information from outside measurement. Among these parameters, measurement of light transmission can be accurately made with a state-of-art spectroscopy facility [4,5] and also a suitable solution method for the forward problem is at present available. However, the development of the reconstruction scheme still remains a challenging problem.

In past decades, various algorithms [6–21] have been introduced to enhance the NIROT technique and great efforts have been steadily directed for more accurate and fast reconstruction algorithms. These first-stage attempts used mostly the diffusion approximation [22] to the equation of radiative transfer (ERT) to solve the forward problem, and gave reasonable results to the specific problems with highly scattering media. It is however known that such diffusion approximation fails for cases of low absorbing and low scattering objects, in particular, when describing light transport in the void-like regions [23,24]. To overcome these limitations, researchers have tried to directly solve the full equation of radiative transfer, rather than use its diffusion approximation, and have further refined it into the time-dependent full ERT version to obtain the maximum information possible [25–31]. This enhancement certainly leads to improved accuracy, as applied to simultaneous reconstruction of absorption and scattering coefficients, as shown in [31]. Nonetheless, this novel modality is still hindered by its difficulty related to practical measurement. Attention is then paid to utilizing a frequency-modulated radiance where not only amplitude but also phase information is available. Indeed, the frequency-domain optical tomography has been steadily pursued by many researchers since 1990.

Their works were however short of general applicability possibly due to the limitations of the diffusion approximation commonly used in their studies [5,7,8,13,15,16]. Thus, since most of the previous studies are focused on the framework of diffusion approximation or on using the time-dependent ERT-based reconstruction problems, there are at present only few studies [32–35] on optical tomography based on the frequency domain equation of radiative transfer and most of them only solved the frequency domain forward problem. In other words, Kui et al. [33–35] is the only group who treated both the forward and inverse problems based on the frequency domain ERT. However several parameters affecting the accuracy of reconstruction are still unsolved. Among them, the choice of a step size coming from the updating scheme of the reconstruction process has been very important since it is closely connected with both the accuracy and the speed of reconstruction. It has also been reported that the conventional choice of the step size common to the unknowns gives inaccurate results as applied to simultaneous retrievals of functional parameters in inverse problems as will be discussed later [36,37]. This explains our motivation for the development of a new frequency-domain ERT-based reconstruction technique improved with the newly proposed method of choosing the step size in the updating scheme.

This work pursues several goals: propose a new reconstruction algorithm generally applicable to the frequency-domain ERT-based optical tomography, evaluate the performance of the enhanced inversion algorithm with numerical experiments, and apply it to the simultaneous estimation of optical properties inside the test medium. To this end, a new solution technique, the sensitivity function-based conjugate gradient method (SFCGM), is described and applied to the inverse problem of simultaneous function estimation of spatially dependent scattering and absorption coefficients in a two-dimensional square medium, and further tested with a cross-talk problem. This method is based on the conjugate gradient method with an adjoint problem formulation [38–40].

In the proposed method, no a priori information is assumed available regarding the forms of unknown functions. The main steps of the algorithm include the followings: forward problem, inverse problem,