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Different shape models for erythrocyte: Light scattering analysis based on the discrete sources method

Elena Eremina[a,*], Jens Hellmers[a], Yuri Eremin[b], Thomas Wriedt[c]

[a]*Universitaet Bremen, Badgasteiner Str. 3, 28359 Bremen, Germany*

[b]*Faculty of Applied Mathematics and Computer Science, Moscow State University, Lenin's Hills, 119992 Moscow, Russian Federation*

[c]*Institut für Werkstofftechnik, Badgasteiner Str. 3, 28359 Bremen, Germany*

Abstract

In practical applications a precise and fast detection of the shape of a single erythrocyte from its scattering characteristics is needed. For this reason detailed investigations of light scattering properties of erythrocyte and their relation to shape is of great interest in recent years. In this paper we analyze light scattering behavior of different shape models of erythrocyte using the discrete sources method. For this we compare scattering results for oblate spheroid, disk-sphere, Cassini-based shape and a shape for a real strainless erythrocyte introduced by Skalak. Numerical results for the scattering indicatrix and the differential cross section by different shape models and its orientations are presented.

Keywords: Erythrocyte; Light scattering; Discrete sources method

1. Introduction

Light scattering by blood cells is recently of great interest both in mathematical modeling and in practical applications. Between other blood cells, the red blood cell—erythrocyte—is the most studied in the last years, because it plays an important role in blood due to its hemoglobin contain. Erythrocytes are the most numerous cells in blood and light scattering by erythrocytes promise to be an appropriate method for detection of some blood diseases. The erythrocyte has an advantage for modeling, as it has no internal structure (like nucleon) and can be modeled as a homogeneous object with a certain refractive index. On the other side light scattering simulation is difficult due to the fact that erythrocyte has a relatively large (with respect to the exciting wavelength) size, which can vary from 4 to 9 μm in diameter and its main shape characteristics: the natural shape of a strainless erythrocyte is a biconcave discoid. But the erythrocyte is surrounded by a thin elastic membrane and can change its form from biconcave to toroidal or to spherical one depending on outward conditions.

In experimental studies and medical diagnostics the precise and fast detection of the shape of erythrocytes by its light scattering is of interest. That is why there is a need for univocal interpretation of experimental

*Corresponding author. Tel: +49 421 218 3583, fax: +49 421 218 5378.
E-mail address: eremina@iwt.uni-bremen.de (E. Eremina).

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results. To provide such accordance between measured scattering parameters of an erythrocyte with its shape a detailed theoretical investigation is necessary. Unfortunately light scattering simulation for biconcave shapes is not an easy task. To simplify this process often models of oblate disks and spheroids are used. In other works the erythrocyte has been modeled based on Cassini ovals, which allow taking into account concavities [1]. There also exist a variety of equations to approximate the real biconcave shape of a strainless erythrocyte [2,3]. An overview can be found in [4].

Recently different methods have been applied to analyze light scattering by a single erythrocyte: finite difference time domain (FDTD) [5,6], discrete dipole approximation (DDA) [4,6], multipole multiple technique (MMP) [7], T-matrix [8]. Some of these methods are not applicable to the real biconcave shape of erythrocyte and others are rather time consuming. Because of this the discrete sources method (DSM) [9] looks very attractive, as it allows making use of the axial symmetry of the particle and polarization of the incident excitation, which sufficiently reduces the time of computations. Besides, the DSM allows computation of scattering for all the incident angles and polarizations at once [10], which is not possible for many other methods.

This is a second paper where the DSM is applied to model erythrocyte. In the first paper [11] the most common non-concave shape models, like disk-sphere and spheroid have been compared to the Skalak model. In this paper we took a Cassini-based biconcave shape in addition to those, taken before. We also calculated the scattering indicatrix, which can be directly measured by detecting devices like scanning flow cytometer [12].

2. Theory outlines and numerical algorithm

Consider scattering in an isotropic homogeneous medium in R^3 of an electromagnetic wave by a local homogeneous penetrable obstacle D_i with a smooth boundary. Let us introduce a cylindrical coordinate system (z, θ, φ) where z is the axis of symmetry of the particle and θ_i is an incident angle with respect to z. Then the mathematical statement of the scattering problem can be formulated in the following form:

$$\nabla \times \mathbf{H}_{e,i} = ik\varepsilon_{e,i}\mathbf{E}_{e,i}, \quad \nabla \times \mathbf{E}_{e,i} = -ik\mu_{e,i}\mathbf{H}_{e,i} \text{ in } D_{e,i}, \quad D_e := R_3/\overline{D_i}, \tag{1}$$

$$\mathbf{n}_P \times (\mathbf{E}_i(P) - \mathbf{E}_e(P)) = \mathbf{n}_P \times \mathbf{E}^0(P), \quad \mathbf{n}_P \times (\mathbf{H}_i - \mathbf{H}_0) = \mathbf{n}_P \times \mathbf{H}^0(P), \quad P \in \partial \mathrm{D} \tag{2}$$

and Silver–Muller radiation condition for the scattered field at infinity.

Here $\{\mathbf{E}^0, \mathbf{H}^0\}$ is an exciting field, $\mathbf{n}_P$ is the outward unit normal vector to $\partial\mathrm{D}$, index e belongs to the external domain D_e, $k = \omega/c$, ε, μ are permittivity and permeability, Im $\varepsilon_e, \mu_e \leq 0$ (time dependence for the fields is chosen as $\exp\{\mathrm{j}\omega t\}$) and the particle surface is smooth enough $\partial\mathrm{D} \subset C^{(1,\alpha)}$. Then the above boundary-value problem is uniquely solvable [13].

The DSM is based on the conception of an approximate solution. The approximate solution is constructed as a finite linear combination of discrete sources (DS): dipoles and multipoles deposited in a supplementary domain inside the particle with certain amplitudes. Usually as such a domain the axis of symmetry of the particle is used. In case of an oblate particle like erythrocyte, disk or oblate spheroid it is not always possible to use the axis of symmetry [14]. For this purpose an analytical continuation to a complex plane is constructed. More detailed information can be found in [11]. The deposition of DS in a complex plane allows reducing calculation errors and time of computations.

In case of P-polarized plane wave the exciting field accepts the following form:

$$\mathbf{E}^0 = (\mathbf{e}_x \cos\theta_0 + \mathbf{e}_z in\theta_0)\exp\{-\mathrm{j}k_e(x\sin\theta_0 - z\cos\theta_0)\},$$

$$\mathbf{H}^0 = -\mathbf{e}_y \cos\theta_0 \exp\{-\mathrm{j}k_e(x\sin\theta_0 - z\cos\theta_0)\},$$

where $k_e = k\sqrt{\varepsilon_e\mu_e}$.

To take into account the polarization of the external excitation we use linear combinations of electrical and magnetic multipoles. For this special vector potentials are used. For the P-polarized wave in a cylindrical