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The output of amplified spontaneous emission lasers

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Abstract

The output of amplified spontaneous emission (ASE) lasers such as X-ray lasers operated without mirrors is calculated exactly for Gaussian and Lorentzian small signal gain profiles by a simple Taylor series expansion. The accuracy of the 'Linford' formula commonly used as an approximation for the output of ASE lasers is evaluated by comparison to our exact solutions. The Linford formula is accurate to better than 10% for intensities produced by a Gaussian gain profile, but requires multiplication by a correction factor of $1/\sqrt{\pi}$, at gain length product greater than 5 for Lorentzian gain profiles.

Keywords: X-ray; Laser; ASE; Plasma

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1. Introduction

The propagation of X-ray laser beams has been investigated in a number of papers [1]. X-ray lasers usually operate without mirrors and the propagation of the beam is determined by the theory of amplified spontaneous emission (ASE) [2]. The small signal gain coefficient g_0 at line centre of an ASE medium, specifies the amplification of intensity achieved. Early studies of X-ray lasers strove to achieve a product of the small signal gain with the length L of the laser medium such that g_0L was at a value where saturation occurred. Saturation of a laser occurs when stimulated emission depletes the population of the upper quantum state and typically occurs for $g_0L \approx 15$ [3].

X-ray lasers have narrow spectral profiles ($\upsilon/\Delta\upsilon \geqslant 10^4$) that cannot be readily spectrally resolved. Nevertheless, Koch et al. [4] constructed a grating spectrometer with resolving power $\upsilon/\Delta\upsilon \approx 35{,}000$ and measured X-ray laser spectral line widths. Other X-ray laser spectral profiles have been resolved using the variation of fringe visibility in interferometry [5,6]. However, most experiments record intensities integrated over the spectral profile of the X-ray laser output.

In the evaluation of experiments, refraction due to spatially varying refractive indices often needs to be considered in the X-ray laser propagation process. Free electrons usually dominate the refractive index and so the refractive index for X-ray lasers is usually close to unity (although an exception has been recently observed [7]). However, due to the long propagation lengths (~1–20 mm) required to achieve high X-ray laser output,

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plasma electron density gradients can refract an X-ray laser beam out of the gain region if the gradients are not reduced by using a pre-pulse to pre-form a plasma with low gradients [8]. If the plasma width approaches the wavelength of the laser, a wave optic treatment of the X-ray laser propagation may be required [9]. However, there are a range of situations in X-ray laser media where a simple ray optic treatment [2,10–12] neglecting refraction is adequate.

In this paper, we consider the X-ray laser medium as comprising a column of length L of uniform gain so that a single ray treatment is appropriate. Linford [10] evaluated an approximate treatment for this situation giving the output intensity as a function of the small signal gain after integrating over the spectral output, but neglecting saturation effects. The formula determined by Linford [10] has been used in more than 100 papers to evaluate the output of X-ray lasers (for example [9,13–15]), but has not been compared to exact solutions in the published literature. Casperson [11] and Pert [2] determined formulae to evaluate the total output intensity of ASE lasers including saturation. The developed formulae relate the peak gain coefficient and the small signal gain coefficient. In the short gain duration situation, the pulse cannot propagate along the gain length during the gain duration [1], so Strati and Tallents [12] evaluated time dependent effects on X-ray laser output associated with a short gain duration.

2. The radiative transfer equations

This paper assumes that a small signal gain coefficient is constant over a cross-section area and along a length L of a plasma column. The X-ray laser propagation can be approximated as occurring in one direction (the z direction), along the plasma length [1]. In a one-dimensional treatment, we calculate the forward irradiance $I_f(\upsilon)$ in a positive z-direction and the backward irradiance $I_b(\upsilon)$ at frequency υ in the negative z-direction by the following [1]:

$$\begin{aligned}\frac{\mathrm{d}I_f(\upsilon)}{\mathrm{d}z} &= G(\upsilon)I_f(\upsilon)+E(\upsilon),\\ \frac{\mathrm{d}I_b(\upsilon)}{\mathrm{d}z} &= G(\upsilon)I_b(\upsilon)+E(\upsilon),\end{aligned} \tag{1}$$

where $G(\upsilon)$ is the gain coefficient and $E(\upsilon)$ the spontaneous emission per unit length.

The gain coefficient $G(\upsilon)$ is related to the small signal gain g_0 at the line centre by

$$G(\upsilon)=\frac{g_0}{1+(I_{av}/I_s)}\frac{f(\upsilon)}{f(0)}, \tag{2}$$

where I_{av} is the X-ray laser irradiance averaged over the line profile, I_s the saturation intensity, and I_{av} is given by

$$I_{av}=\int\frac{f(\upsilon)}{f(0)}I(\upsilon)\,\mathrm{d}\upsilon, \tag{3}$$

where $f(\upsilon)$ is the line profile variation with frequency υ. We consider Lorentzian and Gaussian line profile. Assuming frequency $\upsilon=0$ corresponds to the line centre, a Lorentzian profile [16] is written as

$$f_L(\upsilon)=\frac{\Delta\upsilon_L}{2\pi}\frac{1}{\upsilon^2+(\Delta\upsilon_L/2)^2}, \tag{4}$$

where $\Delta\upsilon_L$ is the full-width at half-maximum of the Lorentzian profile. At line centre

$$f_L(0)=\frac{2}{\pi\Delta\upsilon_L}.$$

A Gaussian profile [17] is written as

$$f_G(\upsilon)=\frac{\sqrt{4\ln 2}}{\Delta\upsilon_G\sqrt{\pi}}\exp\left(-4\ \ln\ 2\frac{\upsilon^2}{(\Delta\upsilon_G)^2}\right), \tag{5}$$