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Milne's problem for anisotropic scattering medium with specular reflecting boundary

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Abstract

The Milne problem is investigated subject to reflecting boundary conditions. The original version of the problem with vacuum boundary condition is generalized assigning, to the surface $x = 0$, a specular reflection coefficient ρ^s $(0 \leqslant \rho^s \leqslant 1)$. Linearly anisotropic case is studied. The integral version of the transport equation solved using trial functions based on Case's eigenvalues and exponential integral function. Solution of the Milne problem is formulated in terms of characteristic parameters such as extrapolated end point, emergent angular distribution and total neutron density. Numerical results for the analytically evaluated parameters are then present. Some of our numerical results are compared with the available published results.

Keywords: Radiative transfer; Milne's problem; Anisotropic scattering; Reflecting boundary

1. Introduction

Milne's problem, which represents an important standard case in the study of neutron diffusion, is completely identical with a problem known in astrophysical literature as "Milne's case". It has been, extensively discussed in connection with the determination of the low of darking at the sun's surface [1]. From the neutron transport point of view, one may consider that,

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the Milne formulation is a reasonable approximation to a situation in which a reactor is buried deep within a shield, and it is desired to determine the angular distribution of neutron leaving the shield [2]. Milne problem with vacuum boundary condition was solved by Placzek and Seidel [1] and Noble [3] using Wiener–Hopf method. Variational approach was employed by Marshak [4] and LeCaine [5]. The formulation of the problem and the solution for isotropic and linearly anisotropic cases were given by McCormick [6], McCormick and Kuščer [7] and Case and Zweifel [2].

The problem was considered for the case of reflecting boundary by Williams [8] using Wiener–Hopf method, by Razi et al. [9] and Abdel Krim and Degheidy [10] using variational methods and later by Atalay using the singular eigenfunction method [11,12].

In this work the integral version of the transport equation was solved using a trial function based on Case's eigenvalues. A linear combination of exponential integral function and Case's eigenvalues has been developed to solve radiation transfer in an absorbing and scattering homogeneous semi-infinite plane parallel medium.

The purpose of the present work is to extend the same technique [13] to include the effect of sources and anisotropic scattering of the medium in solving radiative transfer problems. Some of the numerical results for the extrapolated end point, emergent angular distribution and total density are reported and compared with those using different approaches.

2. Analysis and method of solution

Because it is exactly soluble, Milne's problem has become a venerable yardstick for assessing the performance of approximate treatments of radiative transfer methods. In Milne problem, the general principles are applicable to the distribution of neutrons in a homogeneous source free half-space, through which they are diffusing from a source infinitely deep in the half-space [2]. Far away from the source of particles (photons or neutrons), but still far away from the boundary, the solution falls off exponentially with extrapolation length away from the source. Hence it rises exponentially toward the source.

In this problem we consider the incident flux from $x<0$ which is related to the emerging flux by

$$I(0,\mu) = \rho^{\mathrm{s}} I(0,-\mu), \tag{1}$$

where $I(x,\mu)$ obeys the transport equation as described by

$$\mu \frac{\partial I(x,\mu)}{\partial x} + I(x,\mu) = \frac{\omega}{2}\int_{-1}^{1} p(\mu,\mu') I(x,\mu')\,\mathrm{d}\mu' \tag{2a}$$

$$= \frac{\omega}{2}\{\Phi(x) + 3f\mu q(x)\}, \tag{2b}$$

where $p(\mu,\mu') = (1+3f\mu\mu')$. The physical meaning of Eq. (1) is as follow: a fraction ρ^{s} of the emerging particles are reflected specularly. (f) is the anisotropic term, $f \in [-1,1]$, ω is the single scattering albedo, $0<\omega<1$. Integrating Eq. (2), we obtain the total density

$$\Phi(x) = h(x) + \mathrm{e}^{\upsilon x} \tag{3}$$

and the total current

$$q(x) = h(x) - (1-\omega)\mathrm{e}^{\upsilon x}/\upsilon, \tag{4}$$