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The stochastic radiative transfer equation: Quantum damping, Kirchoffs law and NLTE

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Abstract

A method based on the theory of quantum damping is presented, for deriving a self consistent but approximate form of the quantum transport for photons interacting with a fully ionized electron plasma. Specifically, we propose in this paper a technique of approximately including the effects of background plasma on a photon distribution function without directly solving any kinetic equations for the plasma itself. The result is a quantum Langevin equation for the photon number operator; the quantum radiative transfer equation. A dissipation term appears which is the imaginary part of the dielectric function for an electron gas with photon mediated electron–electron interactions due to absorption and re-emission. It depends only on the initial state of the plasma. A quantum noise operator also appears as a result of spontaneous emission of photons from the electron plasma. The thermal expectation value of this noise operator yields the emissivity which is exactly of the form of the Kirchoff–Planck relation. This non-zero thermal expectation value is a direct consequence of a fluctuation–dissipation relation (FDR).

Keywords: Radiation transport; Non-local thermodynamic equilibrium; Stochastic systems

1. Introduction

In radiative plasmas, the usual governing equation for radiation transport is a semi-classical Boltzmann equation for the specific intensity with sink and source terms coming from emission,

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absorption, and scattering processes. These processes arise from the quantum mechanical interaction of matter and radiation. The standard equation is derived via a heuristic approach where quantum mechanical properties such as photon interference effects are ignored. We call this the ‘‘top–down’’ approach. In the absence of polarization, dispersion effects, and multi-photon effects, the standard radiative transfer equation is

$$\frac{1}{c}\frac{\partial I_\nu(x,\Omega,t)}{\partial t}+(\Omega\bullet\nabla)I_\nu(x,\Omega,t)=j_\nu-\sigma_\nu I_\nu(x,\Omega,t).$$

Here, $I_\nu(x,\Omega,t)$ is the specific intensity defined such that $I_\nu(x,\Omega,t)/c$ is the radiation energy density per unit solid angle per unit frequency. It is proportional to the photon distribution function. The quantity j_ν is the emissivity and σ_ν is the absorptivity. In general, the emissivity and absorptivity are determined by the properties of the atom–radiation interactions and depend on detailed knowledge of the atomic populations and ionization states. As is well known, invoking the assumption of LTE in which the plasma exhibits kinetic, excitation and ionization equilibrium greatly simplifies the problem. For example, a consequence of LTE is Kirchoff's Law; $j_\nu=\sigma_\nu B_\nu(T)$.

A different approach is to derive the radiation transport equation from micro-physical first principles. This is the ‘‘bottom–up’’ approach. Authors, such as Cannon [1], Gelinas and Ott [2], Degl'Innocenti [3], and Graziani [4] begin with the Hamiltonian of quantum electrodynamics (QED). The advantage of this method is that one begins with a formulation containing all degrees of freedom (bound and free electrons, photons, ions, interactions) with a minimum of assumptions. Absorption, emission, and scattering mechanisms arise from the fundamental QED interactions. An equation governing the time evolution of the photon number operator arises from this method. The photon number operator can be cast into an object similar to the specific intensity encountered in the heuristic approach. It is a specific intensity operator formed by using the Klimontovich operator [5]. Extracting a radiation transport equation, similar to the heuristic one discussed above, from the many-body formalism, yields insights into the assumptions underlying the heuristic approach. It also provides a tool to incorporate multi-photon processes, variable refractive index, magnetic fields, etc. Since this method uses a micro-physics approach, it begins with the plasma in NLTE. Additional assumptions have to be placed on the formalism to enforce LTE.

Yet a third way, which we will call the ‘‘Middle-Road’’, has been advocated as a compromise between the heuristic and micro-physics approaches [4,6,7]. The RADIOM model of Busquet [6] is an example as is the work of More et al. [7]. These works attempt to alter the emissivity and opacity in the heuristic transport equation so that NLTE effects can be approximately included. A different approach is to use a rigorous stochastic approach. The difficulty in dealing deterministically with the large number of microscopic elements encountered in a NLTE plasma implies that it might be useful to project all dynamics onto a ‘‘small’’ number of macro-variables [4,8]. The problem, which consists of electrons and photons and their interaction, is decomposed into two types of variables called ‘‘resolved’’ and ‘‘unresolved’’. The resolved or macro-variable degrees of freedom are associated with the photons. The unresolved or micro-variable degrees of freedom are associated with the electron and ions. From non-equilibrium statistical mechanics, this process implies that the resulting dynamics of the macro-variables or photons will be described by a Langevin equation [8,9]. This Langevin equation will have time-history effects. It will involve transfer of information from the macro-to-micro variables (absorption = dissipation) and it will involve transfer of information from micro-to-macro variables (emissivity = fluctuation).