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Nonlinear dynamic compact thermal models by structure-preserving projection

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ABSTRACT

A novel projection-based approach is proposed for constructing compact models of nonlinear heat diffusion problems for electronic components. The method is robust since it preserves the non-linear structure of the heat diffusion equations. It is efficient, since it is constructed by determining few moments of Volterra's series expansions of the solution. It leads to compact models of small state-space dimensions which can be numerically solved at negligible computational cost and to accurate approximations of the whole space-time distribution of temperature rises for all significant waveforms of the injected powers.

1. Introduction

In the last decades many efforts, reported in the literature, have led to distinct procedures for constructing dynamic compact thermal models of heat diffusion problems in electronic components [1–6]. A particularly effective method is the projection-based approach used by the author in his multi-point moment matching technique [7–17]. Such a method leads to robust and accurate dynamic compact thermal models of small state space dimensions, at the low cost of solving few heat diffusion problems in the frequency domain.

Almost all the proposed approaches for constructing compact thermal models apply only to linear thermal problems, in which the thermal properties of materials are assumed to be independent of temperature. However for many heat diffusion problems in electronic components, the temperature dependencies of thermal conductivities on temperature cannot be neglected [18]. In all such cases, in which the dependence of thermal conductivity on temperature has to be taken into account, the most common approach for constructing dynamic compact thermal models is still based on Kirchhoff's transformation [19], by which the nonlinear heat diffusion problem is transformed into an equivalent linear problem. However this method is exact only in very particular situations. In general cases, as shown in [20], it introduces large inaccuracies which cannot be removed.

In this paper a novel approach for constructing compact models of nonlinear heat diffusion problem is proposed, which takes into account the dependencies of thermal conductivity on temperature relevant for electronic components, as modeled in [18]. It stems from a novel reformulation of the nonlinear heat diffusion problem in a form whose projection directly leads a nonlinear compact model. The method is robust, since a novel projection is performed which preserves the nonlinear structure of the reformulated heat diffusion equations. The method is efficient, since the projection space is determined from few moments of Volterra's series expansion of the solution to the problem. Thus it requires only the solution to few linear heat diffusion problems in the frequency domain and does not require computationally costly time-domain solutions to the nonlinear heat diffusion problem. The method also leads to accurate approximations of the whole space-time distribution of temperature rises for all significant waveforms of the injected power, by means of compact models of small state-space dimensions that can be numerically solved at negligible computational cost, as verified by the in-depth investigation of a simple example problem. In this way a novel approach to compact modeling is achieved which extends from the linear to the nonlinear case, most of the advantages of the multi-point moment matching approach, in terms of robustness, efficiency and accuracy. As a first investigation, in this paper only the case of one-port compact thermal models is considered.

The rest of this paper is organized as follows. In Section 2 the nonlinear heat diffusion problem is reformulated. In Section 3 it is shown how nonlinear dynamic compact thermal models can be derived by projecting the reformulated heat diffusion in a way which preserves their nonlinear structure. In Section 4 the projection space is determined by computing the first moments of Volterra's series expansions of the solution to the nonlinear heat

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diffusion problem. Numerical results in Section 5 show the benefits of the proposed approach.

2. Reformulation of the nonlinear heat diffusion problem

A dynamic heat diffusion problem in the spacial domain Ω is ruled by the equation

$$\nabla \cdot (-k(\mathbf{r}, u(\mathbf{r},t))\nabla u(\mathbf{r},t)) + c(\mathbf{r})\frac{\partial u}{\partial t}(\mathbf{r},t) = g(\mathbf{r},t) \tag{1}$$

in which the unknown $u(\mathbf{r},t)$, function of the position vector $\mathbf{r}$ and of the time instant t, is the temperature rise with respect to ambient temperature, due to the power density $g(\mathbf{r},t)$. In order to take into account nonlinear effects, the thermal conductivity $k(\mathbf{r},u(\mathbf{r},t))$ is assumed to depend on $u(\mathbf{r},t)$, in the form suggested in [18]:

$$k(\mathbf{r},u(\mathbf{r},t)) = k(\mathbf{r},0)e^{\mu(\mathbf{r})u(\mathbf{r},t)}. \tag{2}$$

in which $\mu(\mathbf{r})$ is the sensitivity of the thermal conductivity with respect to temperature. The volumetric heat capacity $c(\mathbf{r})$ is independent of $u(\mathbf{r},t)$, as it can be commonly assumed [18]. Eqs. (1) and (2) are completed by conditions on the spacial boundary $\partial\Omega$ of Ω, of outward normal unit vector $\mathbf{n}(\mathbf{r})$. In particular, assuming as it is common, Robin's boundary conditions, it is

$$-k(\mathbf{r},u(\mathbf{r},t))\frac{\partial u}{\partial n}(\mathbf{r},t) = h(\mathbf{r})u(\mathbf{r},t), \tag{3}$$

in which $h(\mathbf{r})$ is the heat exchange coefficient. As usual, homogeneous initial conditions are considered

$$u(\mathbf{r},0) = 0. \tag{4}$$

In preparation to the construction of the nonlinear dynamic compact thermal models, the nonlinear heat diffusion problem is now reformulated in an *equivalent* way. Precisely, by introducing the additional variable

$$\lambda(\mathbf{r},t) = e^{\mu(\mathbf{r})u(\mathbf{r},t)} - 1, \tag{5}$$

(Eqs. (1) and 3) are rewritten in the form

$$c(\mathbf{r})\frac{\partial u}{\partial t}(\mathbf{r},t) + \nabla \cdot (-k(\mathbf{r},0)(1+\lambda(\mathbf{r},t))\nabla u(\mathbf{r},t)) = g(\mathbf{r})P(t), \tag{6}$$

$$-k(\mathbf{r},0)(1+\lambda(\mathbf{r},t))\frac{\partial u}{\partial n}(\mathbf{r},t) = h(\mathbf{r})u(\mathbf{r},t). \tag{7}$$

Besides, by deriving (5) with respect to time, it is written as

$$\frac{\partial \lambda}{\partial t}(\mathbf{r},t) = \mu(\mathbf{r})(1+\lambda(\mathbf{r},t))\frac{\partial u}{\partial t}(\mathbf{r},t) \tag{8}$$

Eq. (8), is *equivalent* to (5), when the initial condition

$$\lambda(\mathbf{r},0) = 0 \tag{9}$$

is introduced. As a result the nonlinear heat diffusion problem (1), (2), (3), and (4) is hereafter reformulated by (4), (6), (7), (8), and (9).

A thermal model is now introduced defining its ports as usual [8]. Thus, limiting to one-port thermal models, for a first exploration, the power density $g(\mathbf{r},t)$ is written in the form

$$g(\mathbf{r},t) = g(\mathbf{r})P(t),$$

in which $P(t)$ is the power generated by the electronic component while

$$T(t) = \int_\Omega g(\mathbf{r})u(\mathbf{r},t)\, d\mathbf{r}, \tag{10}$$

defines the junction temperature rise of the electronic component. As usual, such port allows to connect the thermal model of the electronic component to any model of the heat source. In this way, depending on the model of the heat source, this thermal model can be used for both thermal and electro-thermal analyses of the electronic component. Clearly such a thermal model is *not* compact since it is ruled by the nonlinear heat diffusion equations. A compact thermal model is derived hereinafter.

3. Structure-preserving compact modeling

A non-linear dynamic compact thermal model is here achieved from the heat diffusion equations reformulated in Section 2, by a novel projection approach which preserves their nonlinear structure. To this aim, $u(\mathbf{r},t)$ is approximated in the form

$$u(\mathbf{r},t) = \sum_{j=1}^{\hat{m}_u} u_j(\mathbf{r})\hat{u}_j(t) \tag{11}$$

in which $u_j(\mathbf{r})$, with $j = 1,\ldots,\hat{m}_u$, are a small number of basis functions, which will be determined in Section 4. For the sake of robustness they are assumed to form an orthonormal basis such that

$$\int_\Omega u_i(\mathbf{r})u_j(\mathbf{r})\, d\mathbf{r} = \delta_{ij}, \quad i,j = 1,\ldots,\hat{m}_u,$$

with δ_{ij} being Kroneker's delta function. Similarly $\lambda(\mathbf{r},t)$ is approximated in the form

$$\lambda(\mathbf{r},t) = \sum_{k=1}^{\hat{m}_\lambda} \lambda_k(\mathbf{r})\hat{\lambda}_k(t) \tag{12}$$

in which $\lambda_k(\mathbf{r})$, with $k = 1,\ldots,\hat{m}_\lambda$, are a small number of basis functions, which again will be determined in Section 4. Also, for the sake of robustness, they are assumed to form an orthonormal basis such that

$$\int_\Omega \lambda_i(\mathbf{r})\lambda_k(\mathbf{r})d\mathbf{r} = \delta_{ik}, \quad i,k = 1,\ldots,\hat{m}_\lambda.$$

It is noted that the number $\hat{m}_u$ of the basis functions $u_j(\mathbf{r})$ and the number $\hat{m}_\lambda$ of the basis functions $\lambda_j(\mathbf{r})$ are not necessarily equal.

Multiplying (6) by $u_i(\mathbf{r})$, integrating over Ω, applying the divergence theorem and recalling (7), (11) and (12) result in

$$\sum_{j=1}^{\hat{m}_u} \hat{c}_{ij}\frac{d\hat{u}_j}{dt}(t) + \sum_{j=1}^{\hat{m}_u} \hat{k}^1_{ij}\hat{u}_j(t) + \sum_{j=1}^{\hat{m}_u}\sum_{k=1}^{\hat{m}_\lambda} \hat{k}^2_{ijk}\hat{u}_j(t)\hat{\lambda}_k(t) = \hat{g}_i P(t), \tag{13}$$

in which

$$\hat{c}_{ij} = \int_\Omega c(\mathbf{r})u_i(\mathbf{r})u_j(\mathbf{r})\, d\mathbf{r}, \tag{14}$$

$$\hat{k}^1_{ij} = \int_\Omega k(\mathbf{r},0)\nabla u_i(\mathbf{r}) \cdot \nabla u_j(\mathbf{r})\, d\mathbf{r} + \int_{\partial\Omega} h(\mathbf{r})u_i(\mathbf{r})u_j(\mathbf{r})\, d\mathbf{r},$$
$$\hat{k}^2_{ijk} = \int_\Omega k(\mathbf{r},0)\lambda_k(\mathbf{r})\nabla u_i(\mathbf{r}) \cdot \nabla u_j(\mathbf{r})\, d\mathbf{r},$$
$$\hat{g}_i = \int_\Omega g(\mathbf{r})u_i(\mathbf{r})\, d\mathbf{r}. \tag{15}$$

Similarly multiplying (8) by $\lambda_i(\mathbf{r})$, integrating over Ω and recalling (11) and (12) result in

$$\frac{d\hat{\lambda}_i}{dt}(t) = \sum_{k=1}^{\hat{m}_u} \hat{\mu}^1_{ij}\frac{d\hat{u}_j}{dt}(t) + \sum_{j=1}^{\hat{m}_u}\sum_{k=1}^{\hat{m}_\lambda} \hat{\mu}^2_{ijk}\frac{d\hat{u}_j}{dt}(t)\hat{\lambda}_k(t), \tag{16}$$

in which

$$\hat{\mu}^1_{ij} = \int_\Omega \mu(\mathbf{r})\lambda_i(\mathbf{r})u_j(\mathbf{r})\, d\mathbf{r},$$
$$\hat{\mu}^2_{ijk} = \int_\Omega \mu(\mathbf{r})\lambda_i(\mathbf{r})\lambda_k(\mathbf{r})u_j(\mathbf{r})\, d\mathbf{r}.$$

Also, from (4) and (9) initial conditions for the compact model follow

$$\hat{u}_i(t) = 0, \quad i = 1,\ldots,\hat{m}_u,$$
$$\hat{\lambda}_k(t) = 0, \quad k = 1,\ldots,\hat{m}_\lambda.$$