Microelectronics Journal 45 (2014) 1777–1785

Contents lists available at ScienceDirect

Microelectronics Journal

journal homepage: www.elsevier.com/locate/mejo

# Compact electro-thermal models of interconnects

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ARTICLE INFO

*Article history:*
Received 30 December 2013
Received in revised form
14 May 2014
Accepted 14 May 2014
Available online 7 June 2014

*Keywords:*
Interconnects
Compact Electro-thermal Model
Nonlinear Model Order Reduction
Volterra's series

ABSTRACT

A novel projection-based approach is proposed for constructing compact models of the electro-thermal problems for interconnects. The method is robust since it preserves the non-linear structure of the equations. It is efficient, since it is constructed by determining few moments of the Volterra's series expansions of the solution. It leads to compact models of small state-space dimensions which can be numerically solved at negligible computational cost and to accurate approximations of the whole space-time distribution of voltages, currents and temperature rises within the interconnects for all significant waveforms of the injected powers.



## 1. Introduction

According to the predictions of the International Technology Roadmap for Semiconductors [1], the realization of efficient interconnects is a challenging problem along the route towards nanoelectronics, since they are required to satisfy stringent requirements in terms of electrical and thermal behaviors. In fact the scaling of very-large-scale integration structures implies increasing current densities that result in greater Joule heating and in greater temperature rises. These thermal effects introduce a strong degradation of the electrical speed performance [2]. As a result, interconnects require accurate electro-thermal models [3,4]. The simplest models, commonly adopted, are electrical transmission lines coupled to thermal transmission lines [5]. The resulting models, either spatially distributed or lumped, involve high computational burden for their numerical solution. Electro-thermal models of reduced complexity would thus be crucial in applications. Unlike compact electro-thermal models of electronic devices, a compact electro-thermal model of an interconnect cannot be straightforwardly obtained by coupling a compact model of the electric problem to a compact model of the thermal problem [6–29]. Instead a compact model of the whole electro-thermal problem has to be constructed. Such a problem is however nonlinear so that, as far as the author knows, no results are reported in the literature.

In this paper, a novel projection-based approach is proposed for constructing compact electro-thermal models of interconnects. The method is robust, since a novel projection is performed which preserves the nonlinear structure of the electro-thermal equations. The method is efficient, since the projection space is determined from few moments of Volterra's series expansion of the solution to the problem. Thus it requires only the solution to few linear electric and thermal problems in the frequency domain and does not requires computationally costly time-domain solutions to the nonlinear electro-thermal problem. The method also leads to accurate approximations of the whole space-time distribution of voltage, current and temperature rises in the interconnects for all significant waveforms of the injected power, by means of compact models of small state-space dimensions that can be numerically solved at negligible computational cost, as verified by the in-depth investigation of a simple example problem. In this way a novel approach to compact modeling is achieved which extends to electro-thermal modeling most of the advantages for the electrical modeling of interconnects, in terms of robustness, efficiency and accuracy. As a first investigation, in this paper the case of one-port interconnects modeled by coupled electrical and thermal transmission lines is considered.

The rest of this paper is organized as follows. In Section 2 the electro-thermal problem for an interconnect is formulated. In Section 3 it is shown how nonlinear dynamic compact electro-thermal models can be derived by projecting the equations in a way which preserves their nonlinear structure. In Section 4 the projection space is determined by computing the first moments of Volterra's series expansions of the solution to the electro-thermal problem. In Section 5 it is shown how this choice of the projection space leads to compact electro-thermal models which match the first moments of Volterra's series expansions of the solution to the

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http://dx.doi.org/10.1016/j.mejo.2014.05.012
0026-2692/

electro-thermal problem. Numerical results in Section 6 show the benefits of the proposed approach.

## 2. Formulation of the electro-thermal problem

An interconnect network is assumed to be modeled by an electric transmission line coupled to a thermal transmission line, as shown in Fig. 1. The equations of the electrical transmission line, for the sake of simplicity assumed of RC type, can be written in the form

$$c(x)\frac{\partial v}{\partial t}(x,t)+\frac{\partial i}{\partial x}(x,t)=0, \quad (1)$$

$$\frac{\partial v}{\partial x}(x,t)+r(x,u(x,t))i(x,t)=0, \quad (2)$$

in which $v(x,t)$ and $i(x,t)$ are respectively the voltage and the current at time instant $t$ and position $0\leqslant x\leqslant L$, with $L$ being the length of the transmission line. The electric capacitance per unit of length is $c(x)$ and the electric resistance per unit of length is $r(x,u(x,t))$, which is assumed to be dependent on the temperature increment $u(x,t)$ with respect to the substrate temperature. Such dependence is written in the common form [5]:

$$r(x,u(x,t))=r(x,0)(1+\mu(x)u(x,t)). \quad (3)$$

As a first investigation, one-port interconnects are considered. Thus the current $i(0,t)$ is set as the current $I(t)$ injected at the port and $v(L,t)$ is set to zero. The voltage $v(0,t)$ at the port at which current $I(t)$ is injected is the port voltage $V(t)$. Homogeneous initial conditions are assumed for $v(x,0)$.

As it is usual [5], the temperature rise distribution $u(x,t)$ within the electric transmission line is modeled, by a thermal transmission line, ruled by equations

$$m(x)\frac{\partial u}{\partial t}(x,t)+\frac{\partial}{\partial x}\left(-k_t(x)\frac{\partial u}{\partial x}(x,t)\right)+k_n(x)u(x,t)=g(x,t), \quad (4)$$

in which the thermal capacitance per unit of length is $m(x)$ and the transversal and normal thermal conductances per unit of length are respectively $k_t(x)$ and $k_n(t)$. The power density $g(x,t)$, due to the Joule heating in the electric transmission line, can be written in the form:

$$g(x,t)=-\frac{\partial v}{\partial x}(x,t)i(x,t). \quad (5)$$

The equations of the thermal transmission line are completed by boundary conditions, assumed of Robin's type,

$$-k_t(0)\frac{\partial u}{\partial x}(0,t)=h(0)u(0,t) \quad (6)$$

$$k_t(L)\frac{\partial u}{\partial x}(L,t)=h(L)u(L,t) \quad (7)$$

and by homogeneous initial conditions for $u(x,0)$.

## 3. Structure-preserving compact modeling

The equations formulated in Section 2 provide a spatially distributed electro-thermal model of an interconnect. A compact electro-thermal model of such equations is here achieved by a novel projection approach, which preserves their nonlinear structure. To this aim, $v(x,t)$ is approximated in the form

$$v(x,t)=\sum_{i=1}^{\hat{m}_v} v_i(x)\hat{v}_i(t), \quad (8)$$

in which $v_i(x)$, with $i=1,\ldots,\hat{m}_v$, are a small number of basis functions, which will be determined in Section 4. For the sake of

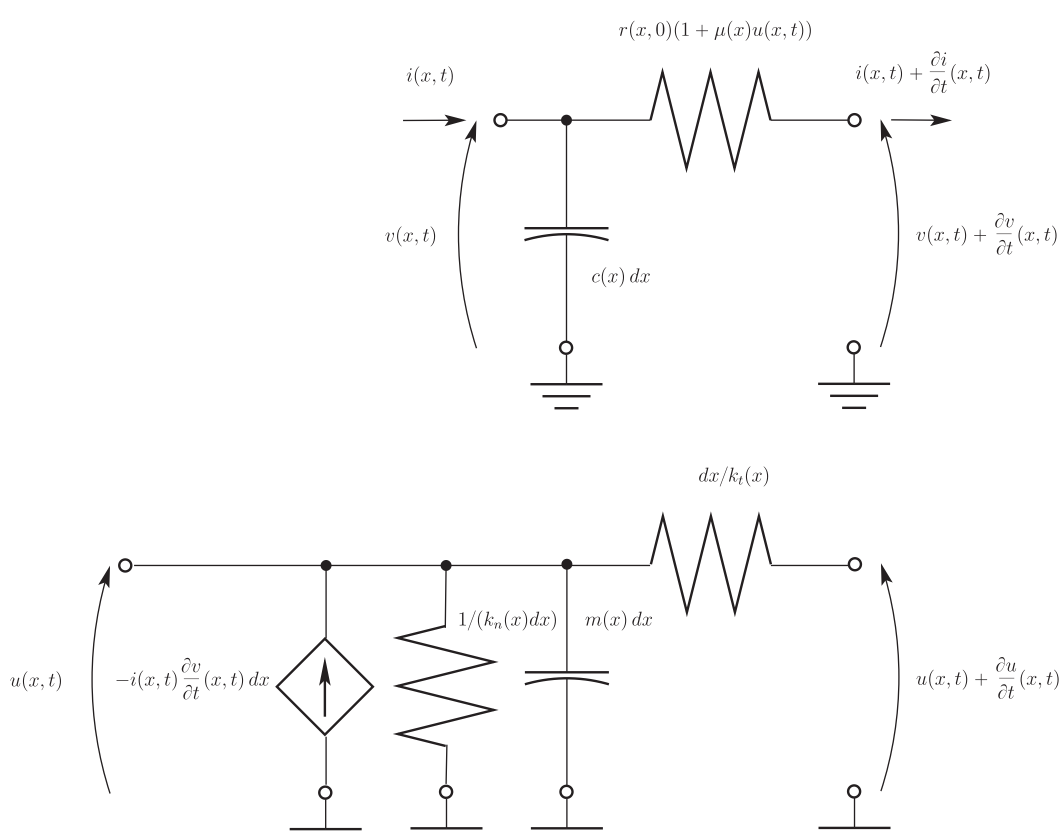


**Fig. 1.** Coupled electric and thermal transmission lines modeling an interconnect.