Composites Communications 6 (2017) 52–58

Contents lists available at ScienceDirect

Composites Communications

journal homepage: www.elsevier.com/locate/coco

Modified thermal resistance networks model for transverse thermal conductivity of unidirectional fiber composite

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ARTICLE INFO

Keywords:
Fibres
Polymer-matrix composites
Thermal conductivity
Thermal-electrical analogy technique

ABSTRACT

In this paper, a facile method based on thermal-electrical analogy technique was developed to solve the heat flow transfer behavior in unidirectional fiber reinforced polymer composite. A modulating thermal resistance R_{md}, correcting the heat flux, was introduced in the thermal resistance network to analyze two-dimensional square arrayed square fiber model of composite for its effective transverse thermal conductivity. The result of present method showed a better agreement with that of finite element method (FEM) than existed parallel or series thermal resistance network. Around this consideration, the composite reinforced by fiber with circular-cross section was also modeled to evaluate the transverse thermal conductivity, which still exhibit well consistent with experimental data. It indicates that the proposed method is enough accurate and effective for evaluating the thermal conductivity of unidirectional fiber composite and provides a facile approach to understand the complicated heat flow transfer behavior in composites.

1. Introduction

Thermal conductivity is one of the important property of composite materials and it has been studied for years [1–3]. It is often desired for composite materials through structural and componential design. In many cases, however, it is difficult to obtain required thermal conductivity of composite materials experimentally because of the complex preparation process. Additionally, direct measurement of the thermal conductivity of fiber is difficult due to the nature of fiber and measurement techniques, it is desirable to predict the longitudinal and radial thermal conductivities of fiber by using the composite and resin thermal properties in models [4]. Thus, many books and articles have introduced various theoretical and numerical methods to predict the thermal conductivity of the composite materials with different style, such as unidirectional and random arrangement fiber and porous composite materials [5–10], and they always attempt to obtain an accurate prediction of effective thermal conductivity for the purpose of well-designing materials.

As the ever-increasing computing technique has been developed, scientists have been paying more and more attention to the numerical methods for the studies [11–15]. Among these, finite element methods are more suitable for the heterogeneous composite material systems and have been employed in the thermal conductivity of composite [16–20]. FEM has the characteristic of the numerical simulations: not constrained by the limitations of the physical experiments; many variables can be independently changed to check the effect of the parameters; More importantly, numerical methods can provide more accurate results for studies of the physical properties due to the high-efficiency ability of solving local problem such as nonlinear heat transfer boundary value problem [21,22], which is much more difficult for the existed theoretical models.

However, the numerical method including FEM is complicated due to the large number of data processing compared to the theoretical method. Therein, it is significant to find out a simple theoretical method to accurately analyze the thermal conduction. Thermal-electrical analogy technique (TEAT) has been considered very useful and fast solution technique for steady and transient 2D multi domain conductive problems, especially for composite materials [1,23,24]. For instance, Federico Scarpa and his coworker obtained an acceptable level of accuracy by TEAT that fully map the admissible current ratings in flat

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http://dx.doi.org/10.1016/j.coco.2017.09.002
Received 23 August 2017; Received in revised form 15 September 2017; Accepted 18 September 2017

cables for industrial applications [24]. Recently, the hemp fiber composite was analyzed through solving a flexible model in representative unit cell originally formulated by Kirkpatrick and the result shows good agreement with FEM [8]. These models are more reasonable than the series and parallel models that only give the lower and upper bounds of the effective thermal conductivity of composites [8]. However, it should be noted that empirical constant have been employed in such models, which have no physical meaning and makes more difficult to understanding the effect of microstructure of materials on the thermal conductivity and further achieving its successful application in many engineering field [25]. Moreover, most of the models based on TEAT exhibit the “lumped” feature but the distribution of heat flow is nonlinear in heterogeneous material mainly embodied in the local heat flow alteration near the interfaces among components, which leads to the significant uncertainty on the evaluation results [26]. Till now, hence, the challenge still maintains on the accurate and simple evaluation method for the real application such as pore materials and composite materials.

The aim of present work is to solve the heat flow transfer in the unidirectional square fiber-polymer composite by settling a simper and accurate model based on thermal-electrical analogy method. To achieve this task, firstly, two-dimensional square arrayed square fiber model (TSS model) is settled attempting to illustrate a simple and reasonable idea; secondly, a modulating thermal resistance R_{md} is employed in thermal resistance network and solved by a liner Equations set; Finally, a good agreement between the results of present method and Finite Element Method is shown in the Figures with the error analysis among methods. It might motivate engineers to effectively study the heat flow transfer behavior in different composites through modified thermal resistance networks.

2. Calculation

2.1. Heat flow transfer in unidirectional fiber composite

Based on parallel and series thermal resistance networks presented in Figs. S1 and S2, the unidirectional square fiber composite was employed and modeled as the two-dimensional square arrayed square fiber (TSS) model unit cell with the inclusion fiber (bray square) embedded in the matrix (white square) shown in Fig. 1. In this case, constituents 1 and 2 stand for fiber and matrix, respectively. To evaluate the effective transverse thermal conductivity of TSS model, the unit cell was separated to four parts I, II, III and IV with the effective thermal resistance R_I, R_{II}, R_{III} and R_{IV} given in Eq. (1).

$$\begin{cases} R_I = \frac{a_f}{K_m(a_m-a_f)} = \frac{V_f^{\frac{1}{2}}}{K_m\left(1-V_f^{\frac{1}{2}}\right)},\ R_{III} = \frac{1}{K_m} \\ R_{II} = \frac{1}{K_f} = \frac{1}{\beta K_m},\ R_{IV} = \frac{a_m-a_f}{K_m a_f} = \frac{1-V_f^{\frac{1}{2}}}{K_m V_f^{\frac{1}{2}}} \end{cases} \tag{1}$$

where V_f, K_f and K_m is the volume percent of fiber, the transverse thermal conductivity of fiber and matrix, respectively. β is the ratio of K_f to K_m.

Thermal resistance networks of TSS model unit was normally arranged to series-parallel and parallel-series thermal resistance networks. Series-parallel resistance networks means thermal resistance of series branches R_I & R_{III} and R_{II} & R_{IV} are connected in parallel, which is called SPR network (SPRN) (see Fig. 1(d)). The thermal conductivity evaluated from this network is expressed in Eq. (2) and employed in many literatures [5,23,27].

$$K^{sp} = \frac{1}{R^{sp}} = \frac{1}{R_I+R_{III}} + \frac{1}{R_{II}+R_{IV}} = K_m\cdot\frac{\left(V_f^{\frac{1}{2}}-V_f\right)(K_m-K_f)+K_f}{K_f+V_f^{\frac{1}{2}}(K_m-K_f)} \tag{2}$$

In the assumption of parallel series resistance network (see Fig. 1(e)), the thermal resistance of parallel branch R_I & R_{II} is connected with that of R_{III} & R_{IV} in series with the abbreviation of PSR network (PSRN). The corresponding thermal conductivity was given in Eq. (3) (called PSRN method). This method has been utilized in many reports [21,28,29].

$$K^{ps} = \frac{1}{R^{ps}} = \left(\left(\frac{1}{R_I}+\frac{1}{R_{II}}\right)^{-1} + \left(\frac{1}{R_{III}}+\frac{1}{R_{IV}}\right)^{-1}\right)^{-1} = K_m\cdot\frac{\left(1-V_f^{\frac{1}{2}}\right)K_m + K_f V_f^{\frac{1}{2}}}{\left(V_f^{\frac{1}{2}}-V_f\right)(K_f-K_m)} \tag{3}$$

Obviously, Eqs. (2) and (3) are the function of V_f, β and K_m. They are completely different to each other. For TSS model shown in Fig. 1(a), it is assumed that no any heat transfer exchanging between layers in SPR network in Fig. 1(d). It can be seen that $Q_I^{sp} = Q_{III}^{sp}$ and $Q_{II}^{sp} = Q_{IV}^{sp}$.

If PSR network is applied for the composite model, some heat flow exchanging between layers becomes to emerge as shown in Fig. 1(b), from which one can see $q_I^{ps} = \frac{Q_I^{ps}}{a_m-a_f} = q_{II}^{ps} = \frac{Q_{II}^{ps}}{a_f}$, and $q_{III}^{ps} = \frac{Q_{III}^{ps}}{a_m-a_f} = q_{IV}^{ps} = \frac{Q_{IV}^{ps}}{a_f}$. Where a_m and a_f is the size parameters of composite walls. Actually, heat flow redistributes after passing parts I and II (i.e. line AB). The heat flow exchanged between different parts is smaller than that in Fig. 1(b) and larger than that in Fig. 1(a). In other words, the actual heat flow transfer neither presents zero-exchanging behavior between layers (SPR network) ($Q_I^{sp} \neq Q_{III}^{sp}$ and $Q_{II}^{sp} \neq Q_{IV}^{sp}$) nor redistributes so uniform that the heat flux in parts III and IV are same to each other (PSR network) ($q_{III}^{ps} = \frac{Q_{III}^{ps}}{a_m-a_f} \neq q_{IV}^{ps} = \frac{Q_{IV}^{ps}}{a_f}$).

2.2. Modified thermal resistance network method (MRN)

To solve above problem, a medium value of heat flow transfer exchanging (see Fig. 1(c)) between parts should be found because the distribution of temperature and heat flow in composite is gradient rather than that in Fig. 1(a) and (b), which is testified in Fig. S3. In present study, a modulating thermal resistance R_{md} is defined on behalf of the actual limited heat flow exchanging in TSS model and introduced into the modified thermal resistance network (MRN) shown in Fig. 1(f).

In Fig. 1(f), the high temperature side is held at T_h, and the low temperature side is held at $T_l = 0$. The temperature decreases to T_I and T_{II} after heat flow transfer through R_I and R_{II}, respectively. Q_I, Q_{II}, Q_{III}, Q_{IV} and Q_{md} are the corresponding heat flow transfer through R_I, R_{II}, R_{III}, R_{IV}, and R_{md}, respectively. R_e is the effective thermal resistance.

If R_I: $R_{III} > R_{II}$: R_{IV}, in other words, $K_f > K_m$, Eq. (4) can be obtained:

$$\begin{cases} Q_I = Q_{III} - Q_{md} \\ Q_{II} = Q_{md} + Q_{IV} \\ R_I = \frac{T_h - T_I}{Q_I},\ R_{II} = \frac{T_h - T_{II}}{Q_{II}},\ R_{III} = \frac{T_I - T_l}{Q_{III}} \\ R_{IV} = \frac{T_{II} - T_l}{Q_{IV}},\ R_{md} = \frac{T_{II} - T_I}{Q_{md}},\ R_e = \frac{T_h}{Q_I + Q_{II}} \end{cases} \tag{4}$$

Let $T_l = 0$, R_{md} is considered as a given quantity. The effective transverse thermal conductivity is solved in Eq. (5):

$$K_e = R_e^{-1} = \left(\frac{T_h}{Q}\right)^{-1} = \frac{Q_I + Q_{II}}{T_h} = \left(\frac{1}{R_I} + \frac{1}{R_{II}}\right)\left(\frac{1}{R_I}\frac{T_I}{T_h} - \frac{1}{R_{II}}\frac{T_{II}}{T_h}\right) \tag{5}$$

where