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A new approach to the mechanics of fatigue crack growth in metals: Correlation of mean stress (stress ratio) effects using the change in net-section strain energy

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ABSTRACT

An unsolved problem in the mechanics of fatigue crack growth is why mean stress or stress ratio in a fatigue cycle has a profound effect on growth rate when compared in terms of stress intensity factor range (ΔK). Specifically, there has not been any fracture-mechanically-consistent theory to explain the effects of mean stress on fatigue crack growth. In this work, a new and generalized driving force parameter for fatigue crack growth, which effectively incorporates the mean stress or stress ratio effect, is developed from energy principles of solid mechanics. A successful explanation of the effect of mean stress on the growth rate is provided. The driving force is *the accumulated change in net-section strain energy*, which develops as a function of increasing crack length and decreasing net-section size in fatigue. A generalized mechanics analysis, showing how to account for the stress amplitude and the maximum stress of a fatigue cycle in the change in net-section strain energy, is presented. Equivalently, the new crack growth parameter can also be interpreted as the *cumulative work done* by cyclic loading at the crack length at which the crack growth data is determined. Several experimental data, including some historically significant fatigue crack growth data, generated on aluminum, steel and titanium alloys, are used to demonstrate the success of the correlation. As a further validation of the proposed concept, it is shown that the empirical Walker parameter ($\Delta K^{0.5}Kmax^{0.5}$), which had some success in correlating stress ratio effect, is in fact related to the change in net-section strain energy.

1. Introduction

The mean stress level of a fatigue cycle has a profound effect on the growth rates of fatigue cracks in metals. Maddox [1] made an excellent review of this subject on developments prior to 1975. Although a great amount of fatigue crack growth (FCG) data has been generated till date, a physically sound approach to correlate the effect of mean stress on fatigue crack growth is not available. What is meant by "correlation" is that when the mean stress (or stress ratio, R), which is the asymmetry of the cyclic loading, is properly accounted for in terms of an effective driving force parameter for crack growth, then the crack growth data for varying stress ratios will be practically indistinguishable from each other.

Fatigue crack growth correlations, even now, rely on empirical approaches that were proposed several decades ago. Paris, in 1998 [2], stated, "… physical modeling of fatigue crack growth remains unsuccessful for all these forty years …" This statement is still true, but with the revision of years to nearly sixty. The fatigue crack growth data, for the simplest case of constant amplitude loading cycle, are empirically correlated in the form of da/dN versus ΔK graphs ($da/dN = C\Delta K^m$, where a is crack size, N is number of cycles and C and m are constants). This relationship is known as the Paris law [3,4]. A change in the mean stress, at a given ΔK level, usually causes a change in crack growth rate, either small or large, depending on the material. A higher level of mean stress usually increases the FCG rate significantly. The scope of the Paris Law, however, is limited to the correlation of FCG at a constant stress ratio. Empirical modifications to the Paris' law have been proposed to explain the variation of FCG rates with stress ratio. One modification is the Forman's equation [5] in which the crack growth rate is expressed as a function of R and ΔK:

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$$\frac{da}{dN} = \frac{C\Delta K^m}{(1-R)K_c - \Delta K} \quad (1)$$

where K_c is the thickness-dependent fracture toughness. Equation (1) has been shown to express the stress ratio effects in the fatigue crack growth data of 7075-T6 alloy [6]. This equation, however, is also an empirical equation and was constructed by forcing the limiting behavior (that is, in the limit of K tending to fracture toughness, da/dN tends to infinity) on the equation, in an attempt to explain the relative shifts in fatigue crack growth curves with R. However, Pearson [7], noting that the Forman's equation did not work well for low toughness materials, proposed a modification, which placed the denominator in Eq. (1) under the square-root. These modifications have been made to necessarily fit the fatigue crack growth data for various stress ratios, but it is evident that the physical foundations are missing in these approaches. Further, neither the Paris nor the Forman/Pearson equation correlates well with the experimental data in the near-threshold regime of crack growth. This poses a difficulty in predicting fatigue life because majority of life is spent at low-ΔK levels, where the mean stress effects on FCG need to be accurately accounted for. In this regard, Paris himself has criticized [2] the misuse of his equation for life prediction, because of extreme sensitivity of predictions to initial growth rates, that is, the growth behavior in the near-threshold regime. A principal reason is that fatigue crack growth at low ΔK levels is greatly affected by the microstructure [8,9]. There have been many additional variants since the Forman equation–Hoeppner and Krupp [10] in 1975 compiled an extensive list of fatigue crack growth equations, most of which are also empirical approximations in one way or the other. It is clear that the approaches presented so far to characterize the FCG behavior are not physically sound and are not adequate to correlate the entire range of fatigue crack growth data.

In reality, since the K_{max} levels at various mean stresses are different, there should be some combination parameter, which includes both the mean stress (or maximum stress) and the stress range, to properly correlate fatigue crack growth rates. Long time ago, Walker [11] showed that a effective stress intensity parameter of the form: $(\Delta K)^m(K_{max})^{1-m}$ successfully correlated (with m~0.5) the fatigue crack growth data of aluminum alloys obtained at various stress ratios. Interestingly, an equivalent form: $(\Delta\sigma)^m(\sigma_{max})^{1-m}$ was shown to work well explaining the mean stress effect found in the smooth S-N data of the same material. Walker stated that a distinction between crack nucleation and propagation is not necessary in applying this empirical approach. However, this work did not explain why the value of m should be ~0.5, leaving one to wonder whether this value was chosen merely to fit the data. There are also other works [12,13], which have also found satisfactory correlation of the mean stress effect on FCG, empirically, using the Walker approach. Additional empirical approaches such as that by Yuen et al. [14], or Sullivan and Crooker [15] or Parida and Nicholas [16] are also noted. It is obvious that there have been several empirical attempts to correlate the mean-stress effect on FCG. Interestingly, these works seem not to need the crack closure concept for explaining the mean stress effect on crack growth, although several publications on crack closure had appeared in the same time frame of the said publications. It seems that there have been two "camps" attempting to explain the mean stress effect on FCG—one using Walker or alternate parameters and not needing crack closure, and, the other advocating the concept of crack closure.

A different approach to explain the mean stress effects on fatigue crack growth was proposed by Elber [17] in 1971 using the concept of crack closure. This concept suggests that, due to the residual tensile stretch in the plastically deformed crack-wake region, the crack faces contact, and the crack closes at a positive load that is higher than the zero load. He proposed an effective cyclic stress (ΔS_{eff}, given by S_{max}-S_{op}, where S_{max} and S_{op} are maximum and crack opening stresses in a fatigue cycle), or equivalently, an effective stress intensity range (ΔK_{eff}) that drives the crack during the part the crack is open during fatigue cycling. In section 7, a full discussion of the difficulties with the explanations of mean-stress effect, on the basis of crack closure, is presented. The advantages of the present work, which does not rely on crack closure, will be shown in that context.

The objective of this work is to demonstrate the correlation of the mean stress effect on fatigue crack growth rates, on the basis of the change in net-section strain energy in the remaining ligament. The principal novelty here is the generalization of the concept of change in net-section strain energy for any mean stress, on the basis of exact solid mechanics principles. It is then shown that a successful correlation of means stress effect, on the basis of the proposed parameter, can be achieved and without resorting to empirical or crack closure concepts. Specifically, the deduction of net-section cyclic strain energy (ΔC_R) parameter, for any mean stress, is presented using strength-of-materials (SOM) analysis of fracture mechanics specimens. It is demonstrated, using extensive data from center-cracked-tension (CCT) and single-edge-notched-tension (SENT) specimens, that the mean stress effects can be successfully correlated on the basis of ΔC_R. The major contribution of this work is the discovery of the *generality* of the energy-based mechanics approach to correlate fatigue crack growth, regardless of stress cycle asymmetry. Because the proposed driving force parameter is energetically complete in terms of capturing the mechanics that drive the crack growth, it is also shown that crack closure concepts are not needed to explain the mean stress effects on fatigue crack growth. Further, the change in net-section strain energy seems to provide a physical basis for the Walker parameter (($\Delta K^{0.5}Kmax^{0.5}$) which has had some success to correlate the mean stress effect on fatigue crack growth.

2. The concept of the change in net-section strain energy with crack extension

When a fatigue crack extends in a uniformly stressed CCT or SENT specimen, the stress over the net -section (the uncracked section or ligament) increases with crack extension until the point of ligament fracture where the net-section stress level becomes equal to the tensile strength of the material. It then seems, intuitively, that the *change* in the net-section cyclic stress (or cyclic strain energy) that occurs in the remaining section with crack extension, will have something to do with the rate of growth of the crack defining that section. A variation in the asymmetry of the fatigue cycle (due to a change in mean stress), for a given amplitude of loading, can cause significant changes in the net-section cyclic strain energy for a given crack length and for a fixed stress amplitude. Hence, for a proper correlation of mean stress effect on FCG, the change in cyclic strain energy that occurs with crack extension, as governed by the loading cycle asymmetry, needs to be determined quantitatively.

The principal contribution of this work is to show how the change in cyclic strain energy can be determined, for any mean stress, in a fracture-mechanically-consistent manner. In our previous work [18], the change in net-section cyclic strain energy parameter (ΔC_a) was defined for the simplest case of cycling loading (R = 0) and was shown to correlate fatigue crack growth data of several materials very well. That study also provided the fracture mechanics justification for ΔC_a by showing the equivalence of the net-section stress determined by SOM calculation and that