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New approach for the correlation of fatigue crack growth in metals on the basis of the change in net-section strain energy

K.S. Ravi Chandran

Department of Metallurgical Engineering, The University of Utah, 135 South 1460 East Rm. 412, Salt Lake City, UT 84112, USA

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ABSTRACT

A new approach to correlate fatigue crack growth data of metals on the basis of *the change in net-section strain energy* is presented. Extensive experimental data, including some historically significant fatigue crack growth data, generated using center-cracked (CCT) and single-edge notched (SENT) tension specimens, are used to demonstrate this correlation. It is shown that this correlation, quite remarkably, is as strong as that based on the stress intensity factor range (ΔK) in fracture mechanics. The reason for this surprising similarity is explored using the analysis of the net-section stress increase in fracture mechanics specimens. It is found that the actual role of the finite-width-correction factor in fracture mechanics is to create the *stress amplification effect* on the finite specimen, having a given crack length, to produce the same K as that of an infinite specimen. The present work demonstrates that the phenomenon of fatigue crack growth can be easily understood on the basis of the *accumulated change in net-section strain energy* which directly relates to the *accumulated work done* at the loading ends, which determines the rate of fatigue crack growth. It is a simpler and a physical approach for the accurate description of fatigue crack growth behavior of metals. The approach simultaneously validates the use of fracture mechanics parameters (ΔK) for fatigue crack growth, by providing the insight that the physical meaning of stress intensity factor is *embedded* in the change in the net-section strain energy.

1. Introduction

Fatigue crack growth (FCG) behaviors of cracks, under largely elastic loading, are correlated on the basis of stress intensity factor range, ΔK. Paris and co-authors [1,2] first suggested the empirical da/dN-versus-ΔK correlation, leading to the well-known Paris Law ($da/dN = C\Delta K^m$, where a is crack size, N is number of cycles and C and m are constants). The ΔK values for finite specimens are determined using various finite-width-correction factors, which capture the effects of specimen geometries, such that FCG data from different test geometries can be correlated. In the first publication the FCG data for the 2024-T3 aluminum alloy, from three different sources, were shown to be in reasonable agreement when plotted on the basis of ΔK. The ΔK is usually calculated from

$$\Delta K = \Delta\sigma\sqrt{\pi a}\,F(a/W) \quad (1)$$

where $\Delta\sigma$ is the cyclic stress and F(a/W) is the finite-width-correction factor for a specimen having width, W. Since then, Paris law, although empirical, has been used widely to correlate the crack growth rates in fatigue testing and analysis based on ΔK [3]. The key premise of the ΔK-correlation is that, for a given ΔK, a unique fatigue crack growth rate exists for a given material. However, there are several unsettled questions that are quite discomforting with respect to the meaning and use of ΔK in characterizing fatigue crack growth. These questions are elaborated below.

The first question is what is the physical meaning of ΔK. The principal issue, in the context of fatigue, is what *physical change* in the material ahead of the crack tip occurs with an increase or a decrease in ΔK as caused by the changes in applied stress or crack length. Most of solid mechanics parameters, characterizing the state of elastic deformations in solids, are either defined or expressed in terms of physically understandable quantities for the pertaining situation. For example, physically, the elastic modulus of a solid is the stress required to produce unit strain in a material. Bulk modulus is the hydrostatic stress required to produce a unit volumetric or hydrostatic strain. Elastic strain energy per unit volume is the work done to deform a unit volume up to a specific level of strain, which can be written as equal to $\sigma^2/2E$, where σ is

E-mail address: ravi.chandran@utah.edu.

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the applied stress and E is the elastic modulus. Such a straightforward physical interpretation cannot be found for the stress intensity factor range. The fact that the K parameter has odd units (e.g. MPa$\sqrt{m}$) suggests that it might be impossible to construct a physical meaning similar to that for elastic/bulk modulus or elastic strain energy in solid mechanics. This is because $\sqrt{r}$ does not relate to anything physically in the material region ahead of the crack tip.

The second question has to do with whether ΔK is the correct form of driving force to correlate fatigue crack growth on the basis of fracture mechanics. To validate the use of K, the relationship of K to the energy release rate, G, in Griffith's crack theory, ($K = \sqrt{GE}$, where E is elastic modulus), is often invoked. Using this relationship, the cyclic energy release rate to correlate with fatigue crack growth should be written as

$$\Delta G = G_{max} - G_{min} = \frac{K_{max}^2}{E} - \frac{K_{min}^2}{E} \tag{2}$$

When ΔK is used to correlate fatigue crack growth at any stress ratio (R) or mean stress, as commonly done, one is inevitably forced into the interpretation of the relationship between ΔK and ΔG as

$$\Delta K = \sqrt{\Delta G \cdot E} \tag{3}$$

Eq. (3) is correct on the basis of Griffith's fracture theory only for R = 0. However, Eq. (2) is the version that can be broadly validated by Griffith's theory for any R, because the cyclic energy released under cyclic stressing will be proportional to $(K_{max}^2 - K_{min}^2)$ and not to ΔK. Hence, it can be argued that the FCG correlations shown on the basis of ΔK are not fracture mechanically correct, especially for R > 0, because only the amplitude of stress cycles is used to compute ΔK. It is well known that mean stress has a strong effect on fatigue crack growth rates, and the definition of ΔK or its common calculation does not factor the mean K value of cyclic loading. The original justification [1] given for the use of ΔK, following Eq. (1), is that it correlated reasonably the fatigue crack growth data obtained from CCT samples from three different sources for R~0. This may be acceptable when ΔK, as it has been, is used as an empirical parameter, but its physical meaning still remains to be established.

The third question is about the singularity at the crack tip, which is suggested to define the similitude condition for the stress intensity factor. What is meant by *similitude* is that the crack tip stress state, as described by the singular stress distribution, uniquely characterizes the crack tip environment regardless of load or crack size or specimen size or geometry. The definition of stress intensity factor on the basis of the mode-I singular stress field is given [4] as $K_I = \sigma_{yy}\sqrt{2\pi r}$ where σ_{yy} is the normal stress distribution in the plane of the crack, ahead of the crack tip, at a distance r from the crack tip. On this basis it is often suggested that the singularity (the way the stress goes to infinity when crack tip is approached, where $\sigma_{yy} \propto (1/\sqrt{r})$ governs the extension of the crack. However, this is not only this is vague, but it also leads to erroneous deductions. This is because the singularity is present for any non-zero value of K leading to the unrealistic notion, for a brittle material, that the crack tip stress is always infinite for any applied stress. This would suggest, nonphysical aspects such as (i) crack tip extension should occur even below the fracture toughness or threshold level for crack growth and (ii) if this starts to happen, then this should spontaneously trigger further fracturing, leading to the notion that catastrophic fracture can occur at any non-zero stress. This is clearly not the case in real fracture or fatigue experiments, and, the theoretical strength of solids at the crack tip is not infinite either. It is therefore clear that K, or ΔK by implication, is not a parameter that can be easily explained on the basis of the nature of singularity at the crack tip. Further, on the basis of similitude relying on singularity of the stress field, it would be expected that crack growth rates obtained at different crack lengths or specimen geometries should agree for a given value of ΔK. More recent works, however, demonstrate the lack of similitude at low ΔK levels [5] and between the data from different specimen geometries [6]. Even within a given specimen geometry, similitude for a given ΔK, at various crack sizes, was not found [7]. A significant variability in fatigue crack growth rates of large cracks, up to a factor of 5.5 in interlaboratory tests has been found [8] when correlated on the basis of ΔK.

The objective of this work is to find a physical and simpler driving force parameter, without the problems that plague ΔK, to accurately correlate the fatigue crack growth data of metals. It is shown that a new physically defined and easily understandable parameter provides as good a correlation of fatigue crack growth as that provided by the fracture mechanics parameter, ΔK. The new parameter is the *change in net section strain energy (ΔC)*. In our preliminary work [9], it was shown that the change in net-section strain energy (ΔC) correlated extremely well the fatigue crack growth data generated in miniature SENT specimens that were loaded under uniform-displacement end condition. It order to show that this correlation was not merely fortuitous and that it works well for a broad rage of specimens geometries, it is demonstrated here, using some of the historic data sets from center-cracked-tension (CCT) and single-edge-notched-tension (SENT) specimens, that the fatigue crack growth correlation provided by ΔC is almost the same as, and is virtually indistinguishable from, that provided from ΔK.

The principal novelty of the present work is that it presents a complete theoretical justification for the change in net-section strain energy parameter, on the basis of solid mechanics load-displacement relations of specimens with and without cracks. This, in particular, leads to the understanding that the accumulated change in net-section strain energy is in fact equivalent to the *accumulated change in work done* at the loading points. Further, a new approach to determine the net-section stress increase in fracture mechanics is presented, which has also served to explain why the ΔC parameter proposed here should provide a correlation of fatigue crack growth that is as strong as that provided by ΔK. This, interestingly, has led to the novel finding that the finite-width-correction factor used in fracture mechanics is, in effect, a *stress amplification factor* that makes a finite specimen to be fracture mechanically equivalent to an infinite specimen. This interpretation of finite-width-correction-factor is a new point of view in fracture mechanics and it helps here to develop a physical meaning for the stress intensity factor. More importantly, the increase in the *net-section stress* (or strain energy), created by the stress amplification effect of the finite-width-correction factor, is found to be nearly the same as the net-section stress increase (or strain energy) that is determined by the strength-of-materials analysis. The analysis illustrates that the physical meaning of fracture mechanics parameters, K or ΔK, is embedded in the net-section strain energy state, a meaning that is *not obvious* in the conventional treatment of fracture mechanics. Thus, the major contributions of this work are the discovery of a simpler and a physical approach to characterize fatigue crack growth and the demystification of the meaning of stress intensity factor in fracture mechanics.

2. The concept of the change in net-section strain energy with crack extension

In a fatigue crack growth experiment, there is a continuous increase in the stress level of the remaining uncracked section, termed here as the net-section stress, as the crack grows. The increase in net-section stress also causes an increase in the corresponding elastic strain energy in a specimen that is loaded