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A physical model and constitutive equations for complete characterization of S-N fatigue behavior of metals

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ABSTRACT

A physical model and constitutive equations for metal fatigue, to characterize completely the S-N fatigue behavior of structural materials, have been developed on the basis of the macroscopic behavior of fatigue crack growth. The macroscopic behavior that forms the basis is that at any time during fatigue the fractional remaining fatigue life is proportional to the fractional remaining uncracked section size. A crack growth functional has been shown to describe this behavior accurately, and universally, for a wide range of materials, test conditions, and specimen geometries. By integrating this functional and with the introduction of physical boundary conditions, a surprisingly compact constitutive equation for the stress-life (S-N) fatigue behavior is derived. The constitutive equation represents accurately the sigmoidal shape of S-N high cycle fatigue behavior of single crystals and polycrystals, including the asymptotic approach of the fatigue data toward a physical endurance limit stress. Metallurgical fatigue strengthening effects due to pre-strain, alloying and grain refinement have also been shown to be accurately predictable. The equation is then expanded to include the mean-stress effects in various forms, which facilitated the complete prediction of stress-life behavior and fatigue limit, for any mean stress, solely from the S-N behavior of fully reversed fatigue data. A very interesting consequence of this is a new, master constitutive equation capturing the mean stress effect on fatigue behavior. Extensive experimental data, generated at various mean stresses, have been used to validate the present approach.

1. Introduction

Fatigue behavior of materials, both from a scientific and engineering perspective, is commonly presented as the stress-life diagram (S-N curve or diagram, where S is applied stress and N is cycles to failure). There has been a great amount of research on fatigue of materials since Wöhler's first experiments [1], circa 1860–1870,and other such works [2] since then. The experimental characterization of fatigue failures of metals, ever since, has largely remained the same. However, there is still a lack of a physically based constitutive equation that can accurately describe the S-N fatigue behavior of materials over a wide range of failure cycles. The earliest approach to characterize the stress-fatigue-life data is by using Basquin's empirical equation: [3]

$$\sigma = A\left(N_f\right)^B \tag{1}$$

where σ is fatigue stress amplitude and A and B are constants. Basquin's idea was that the S-N data, which appear as nonlinear trends when plotted in terms of linear axes, could be represented as linear lines on a log-log graph, especially to identify any deviation toward long life in data at a low stress—such a deviation indicates the beginning of the appearance of the endurance limit or fatigue limit. However, there are problems with Basquin's equation—when the equation is extrapolated to one-cycle-limit, the tensile strength of the material is not recovered. At infinite cycles, the fatigue strength is zero (because B is negative), which incorrectly suggests the non-existence of an endurance limit in fatigue.

Several difficulties arise in fatigue research and analysis due to the lack of a physically based constitutive description of S-N fatigue behavior of materials. First is in characterizing the effects of microstructure on fatigue, since it is not possible to directly relate the coefficients of Eq. (1) to material properties. without using the empirical constants. More importantly, comprehensive S-N fatigue data, gathered over a very wide range of cycles, show sigmoidal behavior with the ends asymptotically approaching the limiting stress values. Clearly, this form of data cannot be represented by the

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power-function based empirical equations. Also, when an expected mean fatigue behavior of a material cannot be constructed from a physical theory, it becomes very difficult to tell whether the microstructure-induced changes in fatigue life is real or an artifact of scatter in data. There is no doubt, however, that for homogeneous materials, such as single crystals and single phase polycrystals, tight S-N fatigue data, representing well the mean fatigue life over a wide range of cycles, can be experimentally obtained without much difficulty. Several examples are shown later in this work. This means that there ought to be a physical basis and a constitutive equation to describe the sigmoidal behavior of S-N data, but nothing has been found yet in the long history of fatigue research.

The objective of this work is to show that a physically based constitutive equation for S-N metal fatigue behavior can be developed from a mechanistic process of macroscopic crack growth and by imposing the essential physical boundary conditions. A surprisingly compact equation that is flexible enough to describe various forms of S-N fatigue data has been derived. It is shown that the equation also exhibits the correct asymptotic behavior at the extremes: (i) tending to ultimate tensile strength at zero fatigue cycle and (ii) asymptotically reaching the endurance limit stress at infinite cycles. The constitutive equation is shown to describe very well the S-N fatigue data of many materials, including the metallurgical fatigue-strengthening effects. More interestingly, it is demonstrated that the physically based S-N constitutive equation can be expanded to include the effects of means stress or stress ratio, R. This is shown to lead to a master constitutive equation for mean stress effects on fatigue life, using which S-N curves for any mean stress can be readily generated from the S-N curve of fully reversed fatigue.

2. Macroscopic mechanism of fatigue crack growth

We have recently [4] shown that at any point of growth of a fatigue crack through a specimen fatigued at constant stress amplitude, the number of cycles needed to break the remaining unbroken ligament is proportional to the remaining fractional length of the ligament. The nomenclature, as it applies to through cracks in center-cracked-tension (CCT) samples and surface cracks in round bars, is described in Fig. 1. The reasoning here is that *the role of crack growth in fatigue is to reduce the size of the uncracked ligament to that required for monotonic fracture at the maximum stress of the fatigue cycle.* Physically, monotonic fracture occurs at the maximum stress of the last fatigue cycle because the average tensile stress in the ligament has reached the residual strength (defined as the tensile strength of a cracked medium) of the ligament, similar to residual strengths of plates with through cracks [5] or surface cracks [6]. It was then found [4] that there exists a unique functional relating the fractional uncracked section size (1-a/W) and the fractional remaining fatigue life ($1-N/N_f$):

$$1-\frac{a}{W}=\left[1-\frac{N}{N_f}\right]^k \tag{2}$$

where 2a and 2W are the crack length and the specimen width and N is the number of elapsed fatigue cycles corresponding to the crack length and N_f is the fatigue life of a crack-free specimen and k is the parameter characterizing the crack growth behavior. Reconstructed crack-length-fatigue-cycles (a vs. N) data for several materials were shown [4] to follow the functional form (Eq. (2)) quite accurately, even at elastic-plastic and environmentally affected fatigue crack growth conditions and for a broad range of materials. The previous evaluation was limited to relatively thick specimens with plane strain conditions. Here, the functional is also shown (Figs. 2–6) to be valid for other extreme fatigue test conditions of through-crack specimens including plane stress and corrosive test conditions as well as for round bars containing surface cracks. This is briefly discussed in the following.

Figs. 2(a)–4(a) illustrate the raw crack length data as a function of fatigue cycles, for through cracks in CCT specimens, for two aluminum alloy sheets and a thin sheet of maraging steel. In Figs. 2(b)–4(b), the reconstructed growth data is compared with the trends predicted by the crack growth functional (Eq. (2)). These test data represent some of the widest range of environmental and applied stress levels. The motivation is to show that the functional is valid for through cracks under uniformly loaded, and wider ranges of test conditions. The crack growth data [7] of 7075-T6 alloy sheets, obtained under various degrees of corrosion-inhibited condition, are shown in Fig. 2(a). The effect of the various vapor phase inhibitors (sodium tetraethylene diamine, phynylenediamine, phynylenediamine + desiccant, cytosine, guanine and hydrazine) is to reduce the rate of fatigue crack growth to a varying degree—it can be seen from Fig. 2(b) that varying k in the crack growth functional (Eq. (2)) from 0.09 to 0.045 describes very well the effect of environment on crack growth rate. The data in Fig. 3 were generated [8] using center cracks in 3 mm thick sheets of 2024-T3 wrought aluminum alloy, over stress amplitudes ranging from 153 to 300 MPa. Fig. 4 presents the growth data [9] of center cracks in maraging steel sheet (1.2 mm in thickness) fatigue cycled at very high stresses, because the yield strength of the steel is high

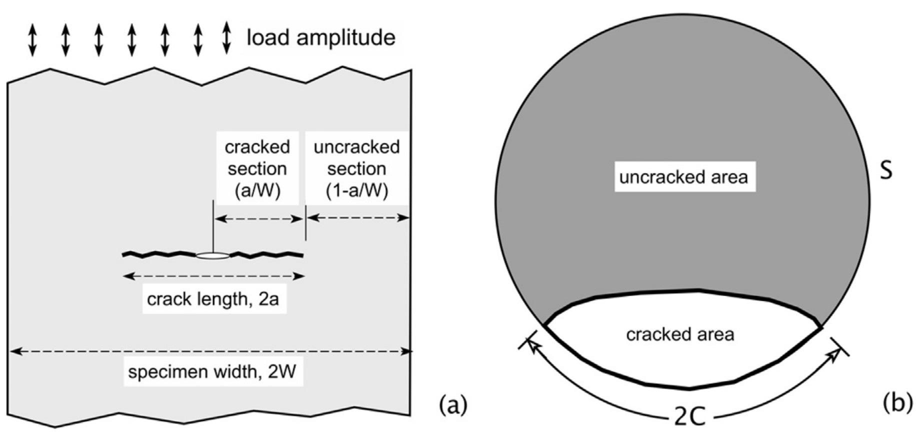

Fig. 1. Schematic of a fatigued CCT specimen defining the normalized cracked and uncracked sections in (a) CCT specimen and (b) the cross-section of a round bar fatigue specimen.