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Theoretical model for predicting uniaxial stress-strain relation by dual conical indentation based on equivalent energy principle

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ABSTRACT

For conical indentation, the strain energy is a function of the semi-vertical cone angle, the indentation depth and the stress-strain relation. According to equivalent energy principle of representative volume elements (RVE) and the classical cavity assumption for material deformation region, the function with dual-parameters about volume and deformation is theoretically derived in the present study. This original equivalent-energy indentation model (EIM) is capable of forward-predicting load-depth relation and reverse-predicting uniaxial stress-strain relation for ductile materials only based on loading part of indentation. Further analyses show that the forward and reverse predicted results from EIM method are in excellent agreement with those by finite element analyses (FEA). Macro conical indentation experiments on five types of metals have been conducted using conventional indenters which are similar to Rockwell sclerometer. Consequently, the stress-strain relations predicted by EIM are quite close to those from standard tensile tests.

1. Introduction

Compared with traditional tension and compression tests, conical indentation is more convenient to get uniaxial stress-strain relationships of materials in the slight-destructive inspection of in-service structures. The elastic solution of conical indentation referring to Boussinesq's solution [1] was developed by Sneddon [2], but the application of the elastic solution was restricted in engineering practice. For the blunted indentation of elastic-perfectly plastic materials, the representative strain ε_r and the cavity assumption were proposed by Tabor [3] and Johnson [4], respectively, which were helpful in describing the characteristic of indentation strain field. In the latest thirty years, the indentation techniques have been developed rapidly for the applications of material testing in MEMS, NEMS, and bioengineering etc. [5–7]. The O&P method to estimate the hardness and elastic modulus of materials was successfully proposed by Oliver and Pharr [8]. After that, there has come up many methods to acquire elastoplastic properties of materials. But until now, all these indentation methods are dependent on a great amount of FEA data and can be mainly summed up in two categories. One directly constructs complicated fitting relations between uniaxial stress-strain response and indented loading-unloading curve [9–21] by using representative strain ε_r and stress σ_r. The other tries to establish complex mathematical regression equations in relation to the ratio of elastic work to plastic work (W_e/W_t), loading curvature C, initial unloading stiffness S and the constitutive parameters of materials (E, σ_y, n) based on dimension analysis [22–25]. These two types of indentation methods are strongly rely on FEA regression, so that testing methods from their complicated formulas are not generally applicable for materials and require high precision of indentation devices. In addition, the forward-predictions of load-depth relation and reverse-predictions of stress-strain relation by these indentation methods are all dependent on indented loading–unloading curve [9–25]. Up to now, there is no research can be found that the parameters of constitutive relationship is only determined by using the single loading curvature C without unloading procedure in the indentation tests.

For materials obeying isotropic, Hollomon power-law, isotropic-hardening and von-Mises criterion, a new indentation model based on equivalent energy principle is established in present study to directly describe the relationship among strain energy, geometrical dimensions, indentation depth and Hollomon parameters, which has dual-parameters, the effective volume coefficient and the

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effective strain coefficient. This equivalent-energy indentation model is hereafter referred to as EIM method. By EIM method, both forward-prediction of load-depth relationship and reverse-prediction of uniaxial stress-strain relationship can be directly determined from the indentation loading curves. The validity and capacity of EIM method have been certified by finite element analyses (FEA) and conical indentation tests on many types of materials.

2. The theoretical model

2.1. The equivalent energy method

It is well-known that the complex plastic deformation behavior of structures can be predicted by uniaxial stress-strain relation of represent volume element (RVE) of material under proportional loading condition. For most of engineering materials, the uniaxial stress-strain relation is commonly described by Hollomon model (see Fig. 1) [26].

$$\sigma = \begin{cases} E\varepsilon & \sigma \le \sigma_y \\ K\varepsilon^n & \sigma \ge \sigma_y \end{cases} \tag{1}$$

where σ is the true stress, ε the total true strain, E the Young's modulus, σ_y the initial yield stress, n the strain hardening exponent, and K the strength coefficient ($K = E^n\sigma_y^{1-n}$). Its dimensionless form is given as

$$\sigma/\sigma_y = \begin{cases} \varepsilon/\varepsilon_y & \sigma \le \sigma_y \\ (\varepsilon/\varepsilon_y)^n & \sigma \ge \sigma_y \end{cases} \tag{2}$$

where ε_y is the initial yield strain and $\varepsilon_y = \sigma_y/E$.

For an arbitrary point (x, y, z) in the indented deformation region suggested by Johnson [4] in Cartesian coordinates (shown in Fig. 2), strain energy U of arbitrary deformation region is expressed as

$$U = \iiint_{\Omega} u(x,y,z)dxdydz \tag{3}$$

where Ω is the effective deformation region and $u(x, y, z)$ is the strain energy density at an arbitrary point within the RVE.

As the strain energy density u is continuous within region Ω, there exists one point M(x_M, y_M, z_M) (see Fig. 2) at which the strain energy density $u_M(x_M, y_M, z_M)$ of RVE is equal to mean result in region Ω. According to integral mean value theorem, it has

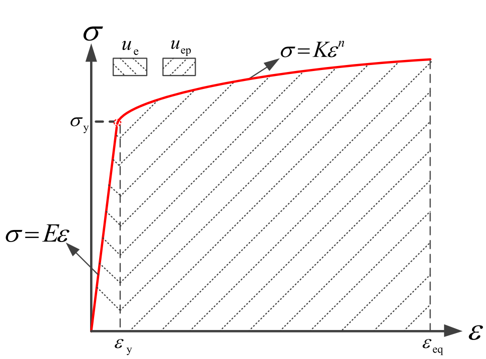

Fig. 1. The uniaxial stress-strain relation and strain energy density of indented materials.

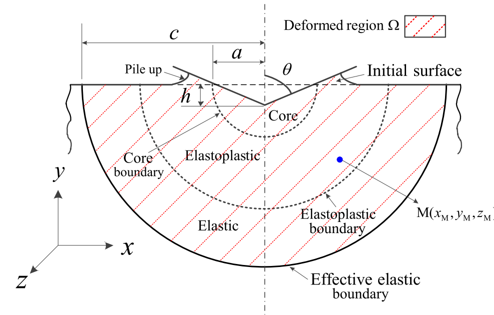

Fig. 2. The deformation region of conical indentation based on cavity assumption.

$$u_M(x_M, y_M, z_M) = \frac{\iiint_{\Omega} u(x,y,z)dxdydz}{V} \tag{4}$$

where V is the total volume of region Ω. In Eq. (4), the point M(x_M, y_M, z_M) is regarded as the equivalent center of strain energy U.

Eq. (4) has established the relation between total strain energy U and the strain energy density u_M at the median point M. Next, a relation between the strain energy density of three-dimensional stress state and that of equivalent stress state can be built. Fig. 3 (a) shows the normal stress-strain state at point M in Ω. For the stress state of RVE at M(x, y, z) in conical indentation, von-Mises criterion gives an equivalent relation between the strain energy density of three-dimensional stress state, u_M and that of equivalent stress state, u_{eq} (see Fig. 3 (a) and (b)),

$$u_M = u_{eq}\big|_{(x_M, y_M, z_M)} \tag{5}$$

Therefore, according to Eqs. (1) and (5), the equivalent strain energy density of RVE around point M is deduced as follows.

$$u_e = \int_0^{\varepsilon_y} \sigma\, d\varepsilon = \frac{E\varepsilon_y^2}{2} = \frac{K\varepsilon_y^{1+n}}{2} \tag{6.1}$$

where u_e is the initial linear-elastic strain energy density (see Fig. 1), then

$$u_{ep} = \int_{\varepsilon_y}^{\varepsilon_{eq}} \sigma\, d\varepsilon = \int_{\varepsilon_y}^{\varepsilon_{eq}} K\varepsilon^n d\varepsilon = \frac{K}{1+n}\left(\varepsilon_{eq}^{1+n} - \varepsilon_y^{1+n}\right) \tag{6.2}$$

where u_{ep} is the elastoplastic strain energy density. Further,

$$u_{eq}\big|_{(x_M, y_M, z_M)} = u_e + u_{ep} = \frac{K}{1+n}\left(\varepsilon_{eq}^{1+n} - \frac{1-n}{2}\varepsilon_y^{1+n}\right) \tag{6.3}$$

where u_{eq} is the total equivalent strain energy density.

Combining Eqs. (4)–(6.3), the total strain energy U is given as

$$U = u_{eq}\big|_{(x_M, y_M, z_M)} V = \frac{KV}{1+n}\left(\varepsilon_{eq}^{1+n} - \frac{1-n}{2}\varepsilon_y^{1+n}\right)\Big|_{(x_M, y_M, z_M)} \tag{7}$$

where u_{eq} is mean - value of equivalent strain energy density in region Ω.

According to work-energy principle, the total work done W by external load is equal to total strain energy U if taking no account of dissipative heat energy from friction etc. i.e.