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On the prediction of the orientation factor and fibre distribution of steel and macro-synthetic fibres for fibre-reinforced concrete

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ABSTRACT

The orientation and distribution of the fibres is decisive in the mechanical behaviour of fibre-reinforced concrete. Several classical models have extensively been used for the case of rigid steel fibres. The increasing interest in structural synthetic fibres that can bend demanded new considerations in this matter. A probabilistic model considering the previous research with stereographical assumptions has been performed allowing the use of fibres that can bend. This paper also provides significant tools for design engineering in order to predict and confirm the number of fibres crossing a vertical surface using fibre reinforced concrete with steel and polyolefin fibres. Additionally, the proposed model coincides with the most accepted values and represents with accuracy the existence of boundaries.

1. Introduction

It is well-known that concrete is weak in tension and that a way of improving this characteristic is adding certain types and amounts of fibres. The latter, which are randomly positioned, might provide some additional residual strength to the concrete element when it bears tensile or flexural stresses. In the last decades, the research and applications of fibres have led to a large variety of shapes [1,2], sizes and constituent materials [3,4]. Moreover, due to all the possibilities previously mentioned the role of the fibres within the concrete element might vary from cracking control or fire spalling prevention, to multifunctional concretes [5–7] that enable applications such as guiding vehicles or heating pavements. Among all these uses one of the most challenging target is the possible substitution (total or partial) of the traditional steel bar reinforcement of concrete in construction industry by certain amounts of fibres.

Steel fibres have been the most common option when using fibres in concrete structural elements. Nowadays there is a wide catalogue of steel fibres with different sizes and shapes which can suit many applications [8]. This common practice has been achieved not only by means of detailed research and application examples but also with the development of test methods [9–11], publications [12,13] and design codes [14–16].

Since fibres are by definition [15,16] randomly added to concrete during the mixing stage, the position and orientation of each isolated fibre depends on the pouring process, formwork geometry and rheological properties of concrete. The positioning of the fibres has been assessed by using stereological tools [17], statistical ones [18] and rheological studies [19,20]. Regarding the orientation of fibres, it should be noted that some models are based on the angles that fibres take in the concrete bulk material. These seem to be appropriate when using methods such as CT-scan [21–23], X-Ray [24–26], optical methods [27–29], methodologies which seek to imitate concrete by using a translucent fluid [30], and others that exploit the electrical properties of concrete in measuring real angles [31–33].

Nevertheless, most of the works performed for this purpose have measured the number of fibres placed in a sawn surface or in the resulting surfaces after fracture tests [34–36]. In such cases, the orientation factor proposed by Krenchel in 1975 [37] enables the use of a factor that offers a coupled value of the orientation and the distribution of the fibres. It provides a powerful comparative factor and allows a better understanding of fracture results. The ease of implementing counting procedures and developing models to

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List of symbols and acronyms

Symbol	
A	Cross section of the sample
A_1	Area representing the base one of probability. In the case of isotropic conditions it corresponds to the area of an equatorial circumference of the sphere of diameter equal to the fibre length
$A_{1-1wall}$	Area representing the base one of probability with the presence of one wall
$A_{1-2walls}$	Area representing the base one of probability with the presence of two walls
A_s	Area of the base of the cone produced in the intersection between the sphere described by the fibre and a vertical plane in a particular case.
A_i	Area of probability of a particular position of the fibre
$A_{i-1wall}$	Area of probability of a particular position of the fibre in the presence of one wall
$A_{i-2walls}$	Area of probability of a particular position of the fibre in the presence of two walls
A_f	Cross section of one fibre
B	Depth of the beam
$c.v.$	Coefficient of variation
D	Height of the beam
d	Diameter of the element
d_e	Equivalent diameter
d_1	Distance of the gravity point of the fibre to a wall
d_2	Distance of the gravity point of the fibre to a second wall
h	Dimensionless distance to a wall as a fraction of the fibre length
h	Specimen height
l	Length of the element
l_f	Fibre length
l_{fr}	Cord of the circumference
n	Number of fibres
$P(\theta_i)$	Probability of orientation factor in a particular case
$P(\theta_{isotropic})$	Mean orientation factor found in the bulk area
R	Radius
th	Theoretical number of fibres
u	Dimensionless distance to a wall as a fraction of the fibre length
V	Volume
V_f	Fibre volumetric fraction
W_f	Dosage of fibres for a volume of concrete

Greek letter symbols	
α	Rotation angle
β	Angle
ρ	Density of the fibres
θ	Orientation factor
θ_i	Probability of occurrence of a particular case
θ_{limit}	Limit angle of the possible positions
$\theta_{isotropic}$	Orientation factor under isotropic conditions
θ_{1wall}	Orientation factor in the presence of one wall
θ_{2wals}	Orientation factor in the presence of two walls
φ	Curvature angle

Acronym	
CT	Computed tomography
FRC	Fibre-reinforced concrete
FR-SCC	Fibre-reinforced self-compacting concrete
LP	Limestone powder
PF	Polyolefin fibre
PFR-SCC	Polyolefin fibre-reinforced self-compacting concrete
PFRC	Polyolefin fibre-reinforced concrete
RC	Steel-bar Reinforced Concrete
SCC	Self-compacting concrete
SF	Steel fibre
SFR-SCC	Steel fibre-reinforced self-compacting concrete
SFRC	Steel fibre-reinforced concrete
VCC	Vibrated conventional concrete

predict the amount of fibres that cross a certain vertical surface has boosted the use of this combination. With the use of the orientation factor (θ), it has been possible to assess the improvements in mechanical properties of SFRC by using self-compacting concrete (SCC) [38] and the effects of several types of vibration on conventional concrete (VCC) [34]. In such a sense, research has confirmed that steel fibres, due to table vibration, tend to orientate along horizontal planes.

The increasing interest on the use of FRC besides the advances on improved synthetic materials has allowed the achievement of polymer fibres that provide concrete with properties and behaviour similar of those of SFRC. Those fibres, essentially polyolefin-based macro-polymer fibres, provide structural capacities with lower dosages in terms of weight. In addition, polyolefin fibres (PF) are chemically stable and reduce the overall cost of the material. The promising results of polyolefin fibre reinforced concrete (PFRC) has been the centre of significant research in recent years, especially focussed in the characterization of the mechanical properties [39–47] and real applications such the use in tunnel linings in which PFRC has been shown as an attractive solution in literature [48–50]. However, there is still a lack of discussion about the orientation factor of PFRC, the effect of fibre-flexibility and length on the relation of the real number of fibres, and the theoretical expected amount of fibres that cross a certain vertical surface. In addition, as macro-synthetic fibres are remarkably different in the physical properties (especially due to lower density and flexibility) when compared with steel fibres, the models that deal with SFRC are questionable.

Moreover, post-cracking behaviour of the FRC has become the reference property. In order to provide trustworthy tools for designers, it is necessary to find predictive models that may allow the prediction of the number and positioning of the fibres placed in the critical sections of a structural piece [38,51,52].

With the aim of addressing this matter, a new model has been built for the theoretical prediction of the number of fibres passing through certain cross section as a reinforcement of concrete and which is adapted for rigid and non-rigid fibres alike. Indeed, the flexible nature of macro-synthetic fibres has been explicitly considered and has encompassed the calculation of the orientation factor of rigid and flexible fibres of different lengths. The model is based on understandable geometrical concepts, solved by numerical integration, and not only fits the most accepted values of the orientation factor but also improves the possibilities of applying counting methods to assess the homogeneity distribution of fibres in concrete elements.